Physics 6107
Statistical Mechanics
Solution of HW 1

2) $98F = 36.67C = 309.67K$.

3) Since this is an adiabatic free expansion there is no change in temperature.

4)

$$c_V = \left(\frac{\partial E}{\partial T}\right)_V = c$$

$$\begin{aligned} c_P &= \left(\frac{dH}{dT}\right)_P = \left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P \\ &= \left(\frac{\partial E}{\partial T}\right)_V + \left(\frac{\partial E}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P \\ &= c + (P + a/V^2)\left(\frac{\partial V}{\partial T}\right)_P = c + \frac{R(P + a/V^2)}{P - a/V^2 + 2ab/V^3} \end{aligned}$$

5) Isothermal Expansion

i) $\triangle E = 0$.
ii) $\triangle W = -\int_{V_0}^{2V_0} pdv = -RT_0 \ln 2$
iii) $\triangle Q = RT_0 \ln 2$
iv) $\triangle H = 0$
v) $\triangle F = -RT_0 \ln 2$.
vi) $\triangle G = -RT_0 \ln 2$

Isobaric Expansion

i) $\triangle E = c_v \triangle T = c_v T_0 = 3/2RT_0$
ii) $\triangle W = -p_0 V0$
iii) $\triangle Q = p_0 V0 + 3/2RT_0 = 5/2RT_0$
iv) $\triangle H = 5/2RT_0$
v) $\triangle F = 3/2RT_0 - T_0 c_p \ln 2 - T_0 S_0$, S_0 being the initial entropy.
vi) $\triangle G = 3/2RT_0 - T_0 c_p \ln 2 - T_0 S_0 + p_0 V_0$.

6) $W_{max} = (1 - T_2/T_1)\,Q = (1 - \frac{293}{673})Q = 0.565Q$

7) The capacitor is at equilibrium under constant potential (constant generalized force), so the enthalpy is minimized.

$$H = E - QV = \frac{Q^2}{2C} - QV$$

The equilibirum value Q_{eq} is the one that minimizes H:

$$\left.\frac{\partial H}{\partial Q}\right|_{Q=Q_{eq}} = 0$$

Solve this equation, we get: $Q_{eq} = CV$.