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NONLINEAR PELTIER EFFECT IN QUANTUM POINT CONTACTS

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A theoretical analysis of the Peltier effect in two-dimensional quantum point contacts, in field-free conditions and under the influence of applied magnetic fields, is presented. It is shown that in the nonlinear regime (finite applied voltage) new peaks in the Peltier coefficient appear leading to violation of Onsager's relation. Oscillations of the Peltier coefficient in a magnetic field are demonstrated. © 1998 Published by Elsevier Science Ltd. All rights reserved

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Ballistic constrictions (point contacts) connecting bulk reservoirs exhibit new properties when their sizes become comparable to the Fermi wavelength of the electrons. In particular, electronic transport in such systems is of quantum (discrete) character, portrayed by conductance quantization (varying in steps of $2e^2/h$, or multiples of the conductance quantum, as a function of the gate voltage, or equivalently the width of the constriction) [1–3] and by the appearance of a peak-like (oscillating) structure of the thermopower coefficient [4–6]. In this paper we study the Peltier effect in such systems, focusing mainly on its nonlinear (finite applied voltage) aspects and its behavior in a magnetic field. We demonstrate the appearance of new peaks in the Peltier coefficient in the nonlinear voltage regime, resulting in violation of Onsager's principle of the symmetry of kinetic coefficients, as well as demonstrate the influence of applied magnetic fields on the Peltier effect.

We consider ballistic electric and thermal transport through a two-dimensional (2D) quantum point contact connecting two bulk reservoirs. A bias voltage V is applied between the reservoirs which are kept at different temperatures T_1 and T_2 . The existence of electrons with different temperatures in the system prevents the establishment of thermal equilibrium.

The thermal transport through the point contact may be described in the entropy current formalism [7] modified in [8] for the Landauer scheme [9]. In this description the electric current, I , and the entropy flow, I_s , [10] are expressed in terms of the equilibrium Fermi functions, f_0 , of the bulk reservoirs and have the forms (we assume the contact to be symmetric about the middle [11])

$$I = \frac{2e}{h} \int dE \left[f_0 \left(\frac{E - eV/2 - \mu_1}{k_B T_1} \right) - f_0 \left(\frac{E + eV/2 - \mu_2}{k_B T_2} \right) \right] \sum T_{nn'}(E), \quad (1)$$

and

$$I_s = \frac{2}{h} \int dE \left[\nu_0 \left(\frac{E - eV/2 - \mu_1}{k_B T_1} \right) - \nu_0 \left(\frac{E + eV/2 - \mu_2}{k_B T_2} \right) \right] \sum T_{nn'}(E). \quad (2)$$

Here the chemical potentials $\mu_i = \mu(T_i)$, $i = 1$ or 2 , are determined by the temperatures of the reservoirs T_i and k_B is the Boltzmann constant; $T_{nn'}$ is the transmission probability for the incident channel n into the conducting channel n' , and the function ν_0 in equation (2) is the entropy density

$$\nu_0(x) = f_0(x) \ln f_0(x) + (1 - f_0(x)) \ln(1 - f_0(x)). \quad (3)$$

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The sums in equations (1) and (2) run over all incident and transmitted channels.

In large (Sharvin's type [12]) point contacts, i.e. for $k_F d \gg 1$, where k_F is the Fermi wave vector and d is the narrowest size of the contact, the sums in equations (1) and (2) may be approximated by kd/π and we obtain the following expressions for the electric current [13]

$$I = -GV^* + K(T_2^2 - T_1^2), \tag{4}$$

and the entropy flow

$$I_s = -K(T_1 + T_2)V^* + \frac{\pi^2 k_B^2}{3e^2} G(T_2 - T_1). \tag{5}$$

Here $V^* = V + (\mu_1 - \mu_2)/e$ is the difference between the electrochemical potentials, $G = (2e^2/h)k_F d/\pi$ is the conductance of the 2D contact (Sharvin's conductance) and $K = (k_B^2 emd)/(6\hbar^3 k_F)$ is the thermoelectric coefficient with m being the electronic mass. The fact that the expressions for I and I_s involve the same thermoelectric coefficient K follows from the Onsager principle. Note that relations (4) and (5) were obtained here beyond the standard formulation of nonequilibrium thermodynamics, since the difference $T_2 - T_1$ was not assumed to be small. The thermoelectric power, ϵ_T , appearing in the

contact can be obtained by setting $I = 0$ in equation (4),

$$\epsilon_T = \frac{\pi^2 k_B^2}{12e\epsilon_F} (T_2^2 - T_1^2), \tag{6}$$

(where ϵ_F is the Fermi energy), which in the linear regime (with respect to temperature difference) leads to the following expression for the thermopower coefficient [13]

$$S = \frac{\pi^2 k_B^2 T}{6e\epsilon_F}. \tag{7}$$

Note the decrease by a factor of 2 compared with the thermopower coefficient of a three-dimensional point contact [7]. The Peltier coefficient is given by $\Pi = ST$.

In the quantum limit, i.e. when the Fermi wave length is of the order of the contact size, one needs to calculate the transmissions coefficients $T_{m'}$ in equations (1) and (2). To this aim we will model the contact by a saddle-shaped potential (the Büttiker model) [14]

$$V = V_0 - \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2, \tag{8}$$

where V_0 is the potential at the saddle point, and the frequencies ω_x and ω_y characterize the constriction shape

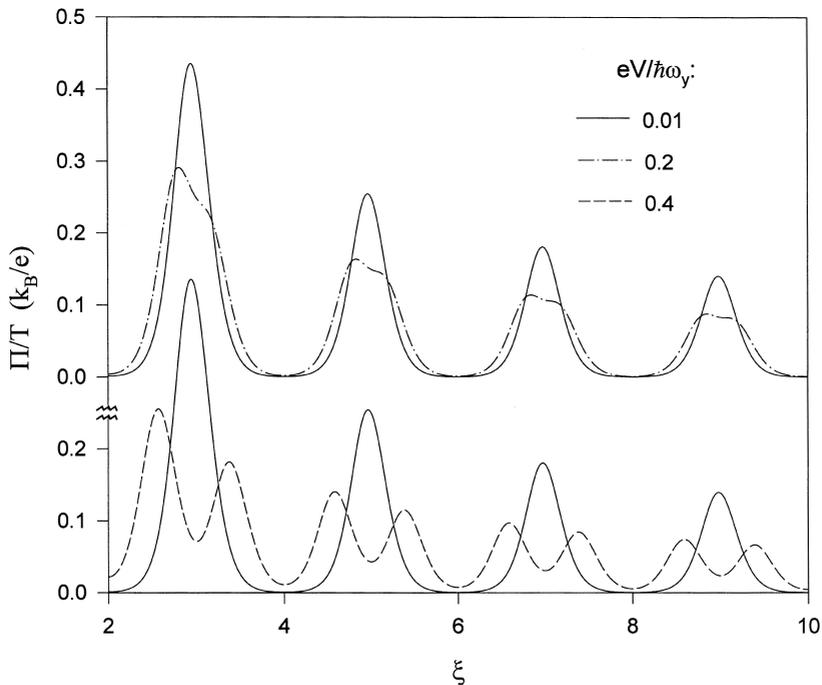


Fig. 1. The Peltier coefficient (Π , in units of $k_B T/e$) of a 2D contact plotted vs the dimensionless parameter $\xi = 2(E - V_0)/\hbar\omega_y$, for $H = 0$, $k_B T/\hbar\omega_y = 0.05$ and $\omega_y/\omega_x = 7$. The different curves correspond to the three marked values of the applied voltage (V in units of $\hbar\omega_y/e$). In the linear regime ($V \rightarrow 0$) the Peltier coefficient (solid line) coincides with the thermopower coefficient according to the Onsager relation $\Pi = ST$.

and the lateral confinement. When a magnetic field is applied in the direction perpendicular to the contact plane, the transmission probabilities have a form [15]

$$T_{m' m} = \delta_{m' m} \{ 1 + \exp[-2\pi(E - V_0 - \hbar\omega_+(n + \frac{1}{2}))/\hbar\omega_-] \}^{-1}, \quad (9)$$

where

$$\omega_{\pm} = \frac{1}{\sqrt{2}} [(\Omega^4 + 4\omega_x^2\omega_y^2)^{1/2} \pm \Omega^2]^{1/2}, \quad (10)$$

$$\Omega^2 = \omega_c^2 + \omega_y^2 - \omega_x^2, \quad (11)$$

and $\omega_c = eH/mc$ is the cyclotron frequency.

We examine first the differential Peltier coefficient of the quantum contact, Π , defined as

$$\Pi = T \left. \frac{\partial I_s}{\partial T} \right|_{T_2=T_1=T}, \quad (12)$$

in a zero magnetic field ($\omega_c = 0$ in equations (10) and (11)). The Peltier coefficient exhibits a peak-like structure with the positions of the peaks coinciding with the positions of the differential conductance steps calculated at the same values of the applied voltages [11]. The variation of the Peltier coefficient as a function of the dimensionless parameter $\xi = 2(E - V_0)/\hbar\omega_y$ is shown in

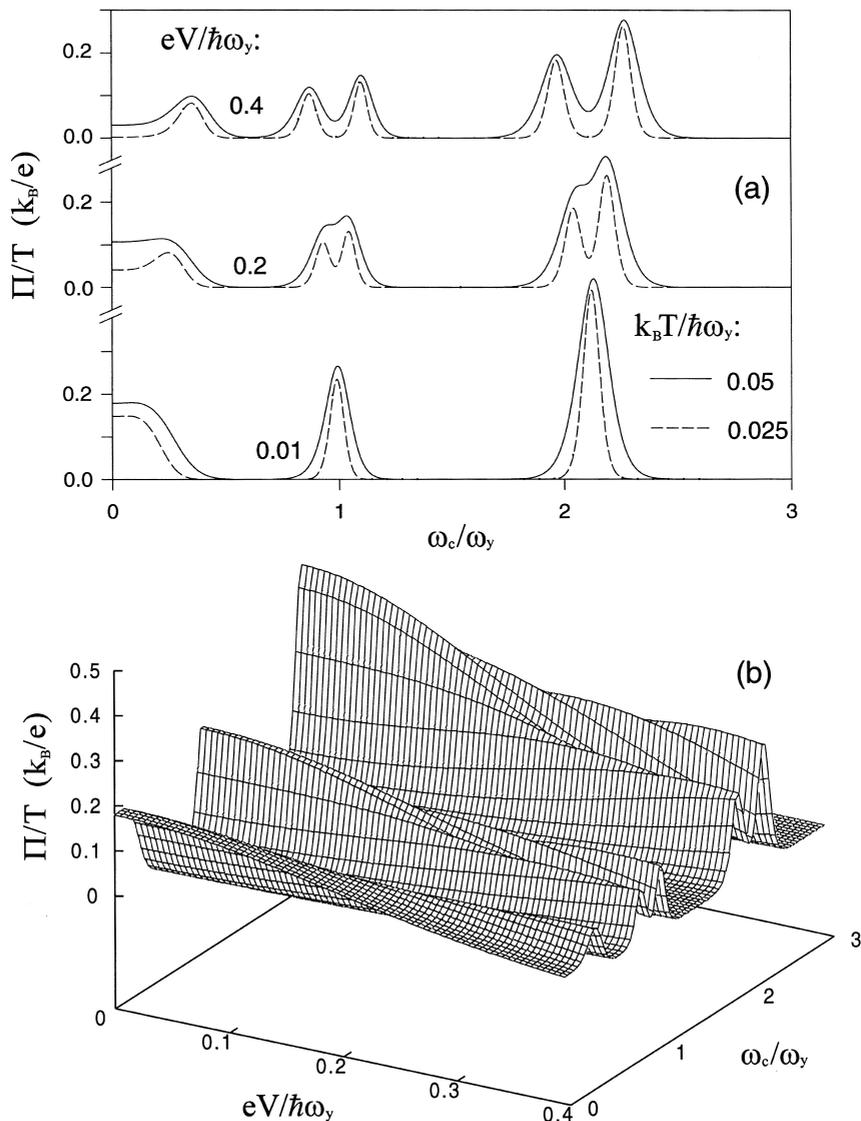


Fig. 2. (a) The Peltier coefficient (Π , in units of $k_B T/e$) of a 2D contact plotted vs the dimensionless cyclotron frequency ω_c/ω_y with $\omega_y/\omega_x = 7$ and $\xi = 7$ (corresponding to four conducting channels for $H = 0$ and $V \rightarrow 0$). Different curves correspond to the marked values of the applied voltage (V in units of $\hbar\omega_y/e$) and in each case for two different temperatures (T , in units of $\hbar\omega_y/k_B$). (b) The Peltier coefficient of the contact plotted vs ω_c/ω_y and $eV/\hbar\omega_y$ for $k_B T/\hbar\omega_y = 0.05$.

Fig. 1 for $k_B T = 0.05 \hbar \omega_y$, $\omega_y/\omega_x = 7$ and for several values of the applied voltage V . In Fig. 1 the Peltier coefficient (in units of $k_B T/e$) plotted for $V \rightarrow 0$ (solid lines in the top and bottom curves) coincides with the thermopower coefficient (the Seebeck coefficient calculated for a similar model in [5]), S , in accordance with the Onsager relation $\Pi = ST$. Increase of the applied voltage leads to the appearance of new peaks in the Peltier coefficient (dash-dotted line in the top and dashed line in the bottom curves, corresponding to the voltages eV equal to $0.2 \hbar \omega_y$ and $0.4 \hbar \omega_y$, respectively) and a consequent violation of the Onsager relation. Note that the origin of the appearance of the new peaks in the Peltier coefficient is that a finite voltage differentiates right- and left-moving electrons leading to the existence

of different effective chemical potentials for opposite moving electrons (see equations (1) and (2)). We note here that the nonlinearity of the Peltier coefficient in quantum contacts appears on an eV -scale of the order of the level spacing between electronic states in the constriction, unlike the case of classical contacts where the parameter characterizing the nonlinearity is eV/ϵ_F .

In a magnetic field the transverse energy levels of the electrons equal to $\hbar \omega_+(n + 1/2)$. The magnetic field shifts the transverse energy levels resulting in the appearance of an oscillating structure of the Peltier coefficient, similar to oscillations of the thermopower coefficient discussed in [5]. In Figs 2(a) and 3(a) we display, for several values of V and the temperature T , the

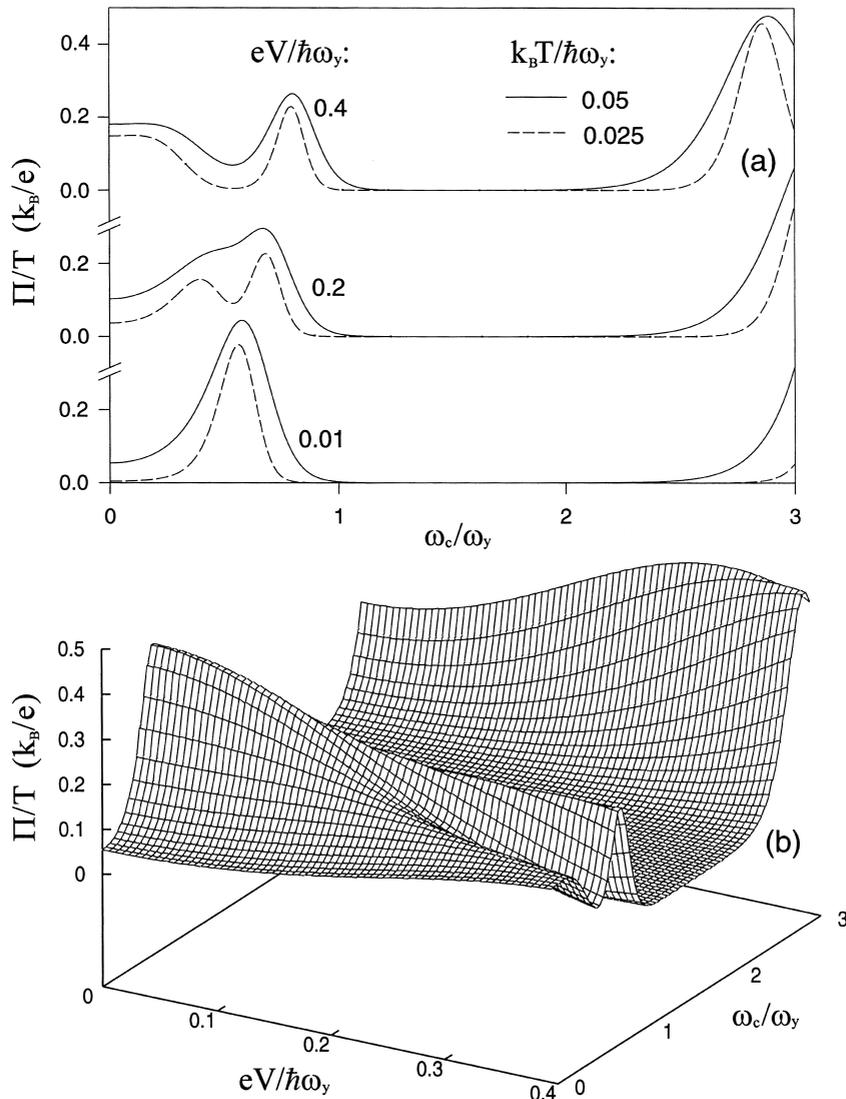


Fig. 3. Same as Fig. 2, but for a 2D contact with $\xi = 3.4$ (corresponding to two conducting channels for $H = 0$ and $V \rightarrow 0$).

dependence of the Peltier coefficient on the dimensionless cyclotron frequency ω_c/ω_y , for a contact with $\omega_y/\omega_x = 7$ and $\xi = 7$ and 3.4, corresponding respectively to four and two conducting channels at $H = 0$ and $V \rightarrow 0$. At $V \rightarrow 0$ the magnetic field dependencies of the Peltier and thermopower coefficients are the same (bottom curve). Increase of the voltage leads to differences in the behavior of the Peltier (upper curves) and thermopower coefficients. The behavior of the Peltier coefficient can be influenced either by an applied magnetic field, an applied voltage or combinations of the two as shown in Figs 2(b) and 3(b) for $k_B T = 0.05\hbar\omega_y$.

The above analysis shows that the Peltier effect in two-dimensional quantum contacts may be influenced and controlled by external parameters such as an applied voltage and a magnetic field. We have demonstrated here theoretically a nonlinear Peltier effect in quantum contacts, exhibited by the appearance of new peaks at finite voltages (deviation from the Onsager relation, Fig. 1), as well as magnetic field induced oscillations of the Peltier coefficient at finite voltages (Figs 2 and 3). Such a behavior of the Peltier coefficient in quantum contacts is due to the influence of the external fields on the spectrum of electronic states in the microconstrictions, allowing one to change and control the number of conducting channels. These effects might be experimentally observed under magnetic fields and applied voltages, such that $\hbar\omega_c$ and eV are of the order of the spacing between the lateral electronic energy levels $\hbar\omega_y$. In contacts created in a 2D electron gas in GaAs/AlGaAs heterostructures, with a typical value of the subband energy level spacings of 2 meV and an effective mass $m^* = 0.067 m_0$ [16] (where m_0 is the free electron mass), magnetic fields of several tesla at helium temperatures should allow observation of the predicted effects.

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