3)

Let T_{eq} the equilibrium temperature

$$\eta = 1 - \frac{T_0}{T_e} = \frac{Wt}{Q_1}$$

$\leftarrow$ work input. (t - ti

$\nwarrow$ heat dumped into the room

at ~~Equilibrium~~ equilibrium, Q_1 = heat leak

$$= t\alpha(T_e - T_0)$$

$$\eta = \frac{W}{\alpha(T_e - T_0)} = \frac{T_e - T_0}{T_e}$$

or $$W = \frac{\alpha}{T_e}(T_e - T_0)^2$$

Solve for T_e one gets

$$T_e = T_0 + \frac{W}{2\alpha} + \sqrt{T_0 \frac{W}{\alpha} + \left(\frac{W}{2\alpha}\right)^2}$$

4) Use the Maxwell relation $\left.\frac{\partial X}{\partial T}\right|_F = \left.\frac{\partial S}{\partial F}\right|_T$

for magnetic system $\left.\frac{\partial M}{\partial T}\right|_H = \left.\frac{\partial S}{\partial H}\right|_T$

$$\Rightarrow \left.\frac{\partial M}{\partial T}\right|_H = -\frac{CH}{T^2} \Rightarrow M = \frac{CH}{T}$$

(boundary condition $M \to 0$ as $T \to \infty$

5)

$$P(x) = \begin{cases} C & 0<x<a \\ 0 & \text{if } 0>x \text{ or } x>a. \end{cases}$$

+ Normalization $\int dx\ P(x) = 1 \Rightarrow C = \frac{1}{a}$.

+ Mean $\langle x \rangle = \int dx\ x\ P(x) = \int_0^a dx\ x\ \frac{1}{a} = \frac{a}{2}$

+ variance $\langle x^2 \rangle = \int_0^a dx\ x^2 \frac{1}{a} = \frac{a^2}{3}$

so $\langle x^2 \rangle_c = \langle x^2 \rangle - \langle x \rangle^2 = \frac{a^2}{3} - \frac{a^2}{4} = \frac{a^2}{12}$

+ Skewness $\langle x^3 \rangle_c = \langle x^3 \rangle - 3\langle x \rangle \langle x^2 \rangle + 2\langle x \rangle^3$

$\langle x^3 \rangle = \int_0^a dx\ x^3 \frac{1}{a} = \frac{a^3}{4}$

$\langle x^3 \rangle_c = \frac{a^3}{4} - 3 \cdot \frac{a}{2} \cdot \frac{a^2}{3} + 2\frac{a^3}{8} = 0$

+ Curtosis $\langle x^4 \rangle_c = \langle x^4 \rangle - 4\langle x^3 \rangle \langle x \rangle - 3\langle x^2 \rangle^2 + 12\langle x^2 \rangle \langle x \rangle^2 - 6\langle x \rangle^4$

$\langle x^4 \rangle = \int_0^a dx\ x^4 \frac{1}{a} = \frac{a^4}{5}$

$\langle x^4 \rangle_c = \frac{a^4}{5} - 4\frac{a^3}{4} \cdot \frac{a}{2} - 3\frac{a^4}{9} + 12\frac{a^2}{3} \cdot \frac{a^2}{4} - 6\frac{a^4}{16}$

$= -\frac{a^4}{120}$

6) a)

$$\Omega(V,N) = \frac{V!}{N!\,(V-N)!}$$

b)

$$S = k_B \ln \Omega = k_B \ln \frac{V!}{N!\,(V-N)!}$$

Stirling's formula $\ln x! \simeq x \ln x - x$ for $x \gg 1$

$$\Rightarrow S \simeq k_B \Big[V\ln V - V - N \ln N + N - (V-N)\ln(V-N) + (V-$$

$$= k_B \Big[V \ln V - N \ln N - (V-N) \ln (V-N) \Big]$$

$$= k_B \Big[-N \ln \frac{N}{V} - (V-N) \ln \frac{V-N}{V} \Big]$$

$$\frac{S}{N} = -k_B \Big[\ln \frac{N}{V} + \left(\frac{V}{N} - 1\right) \ln\left(1 - \frac{N}{V}\right) \Big]$$

$$= -k_B \ln n - k_B\left(\frac{1}{n} - 1\right) \ln (1-n)$$

$$n = \frac{N}{V}\,.$$

$$= -\frac{k_B}{n} \Big[n \ln n + (1-n) \ln (1-n) \Big]$$

c)

$$dS = \frac{1}{T}\, dE + \frac{P}{T}\, dV$$

$$\Rightarrow \frac{P}{T} = \frac{\partial S}{\partial V}\Big|_E = \frac{\partial (S/N)}{\partial (V/N)}\Big|_E =$$

$$= \frac{\partial (S/N)}{\partial (n^{-1})}\Big|_E = -n^2 \frac{\partial (S/N)}{\partial n}\Big|_E$$

$$\frac{P}{T} = n^2 \cdot k_B \left[\frac{1}{n} + \left(-\frac{1}{n^2}\right) \ln(1-n) + \left(\frac{1}{n}-1\right) \frac{-1}{1-n} \right]$$

$$\boxed{\frac{P}{T} = -k_B \ln(1-n)}$$

$$n \to 0 \quad \frac{P}{T} \simeq -k_B \left[(-n) - \frac{(-n)^2}{2} + \dots \right]$$

$$= k_B n + k_B \frac{n^2}{2} + \dots$$

d) at constant ~~potential~~ μ & T, the ~~Gibbs~~ Grand potential is minimized

$$G = E - TS - \mu N$$

$$\frac{\partial G}{\partial N} = 0 \Rightarrow -T \cdot \frac{\partial S}{\partial N} - \mu = 0$$

$$\Rightarrow \frac{\mu}{T} = -\frac{\partial S}{\partial N} = -k_B \left[-\ln\frac{N}{V} - 1 + \ln\frac{V-N}{V} + 1 \right]$$

$$= -k_B \ln \frac{V-N}{N}$$

$$\Rightarrow e^{-\frac{\mu}{k_B T}} = \frac{V-N}{N}$$

$$\Rightarrow N = \frac{V}{1 + e^{-\mu/k_B T}}$$