Physics 6107
Statistical Mechanics
Solution of HW 2

1)

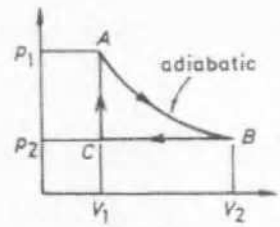

Fig. 1.23.

The work the system does in the cycle is

$$W = \oint p dV = \int_{AB} p dV + p_2(V_1 - V_2) \ .$$

Because AB is adiabatic and an ideal gas has the equations $pV = nkT$ ar
$C_p = C_v + R$, we get

$$\begin{aligned}\int_{AB} p dV &= -\int_{AB} C_v dT = -C_v(T_2 - T_1) \\ &= \frac{1}{1-\gamma}(p_2V_2 - p_1V_1) \ .\end{aligned}$$

During the CA part of the cycle the gas absorbs heat

$$\begin{aligned}Q &= \int_{CA} T dS = \int_{CA} C_v dT = C_v(T_1 - T_2) \\ &= \frac{1}{1-\gamma} V_1(p_2 - p_1) \ .\end{aligned}$$

Hence, the efficiency of the engine is

$$\eta = \frac{W}{Q} = 1 - \gamma \frac{\frac{V_2}{V_1} - 1}{\frac{p_1}{p_2} - 1} \ .$$

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2) Since the total internal energy of the system remains constant, the equilibrium temperature T_f is given by:

$$\frac{3}{2}Nk(T_A+T_B) = \frac{3}{2}.2NkT_f$$
$$\text{i.e.} \qquad T_f = \frac{T_A+T_B}{2}$$

Since entropy is a state function it does not matter how the final state is obtained from the initial one. Thus suppose the wall is diathermal, then the two compartments would first reach thermodynamic equilibrium by having temperature T_f and volume $(V_A+V_B)/2$ each; if the wall is then removed there would be no change in the entropy since there is no entropy of mixing. Thus:

$$\begin{aligned}\triangle S = \triangle S_A + \triangle S_B &= C_v \ln\frac{T_A+T_B}{2T_A} + Nk\ln\frac{V_A+V_B}{2V_A} + \\ &\quad C_v \ln\frac{T_A+T_B}{2T_B} + Nk\ln\frac{V_A+V_B}{2V_B} \\ &= C_v \ln\frac{(T_A+T_B)^2}{4T_AT_B} + Nk\ln\frac{(V_A+V_B)^2}{4V_AV_B}\end{aligned}$$

Recall now that arithmetic mean is greater than geometric mean. Thus the change in the entropy is positive.

4) The mean, second moment and variance of the score of a die is:

$$i) < x > = \frac{1}{6}(1+2+3+4+5+6) = 3.5$$

$$ii) < x^2 > = \frac{1}{6}\sum_{m=1}^{m=6} m^2 = 15.167$$
$$iii) \sigma^2(x) = < x^2 > - < x >^2 = 2.917$$

5) The probability that a six is obtained at the m-th trial is given by:

[probability that a six has not been scored in any of the first $(m-1)$trials] × [a six is scored at the m-th trial]. Thus

$$P(m) = \left(\frac{5}{6}\right)^{m-1}\left(\frac{1}{6}\right)$$

$$< m >= \sum_m m \left(\frac{5}{6}\right)^{m-1} \left(\frac{1}{6}\right) = \frac{1}{6} \sum \frac{d}{dx} x^m |_{x=5/6} = 6$$

6)

$$< f(x) + g(x) >= \int (f(x) + g(x)) P(x) dx = \int f(x) P(x) dx + \int g(x) P(x) dx =< f(x) >$$

7) For a binomial distribution with parameters n, p the probability $p(k)$ of getting k successful trials of an event is given by:

$$p(k) = \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

where p is the probability that a trial is successful and q is probability that it is not,i.e., $q = 1 - p$. Thus:

$$< k >= \sum_{k=0}^{k=n} k \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

$$= p \frac{\partial}{\partial p} \left(\sum_k \frac{n!}{k!(n-k)!} p^k q^{n-k} \right) = np$$

Similarly

$$< k^2 >= p^2 \frac{\partial^2}{\partial p^2} \left(\sum_k \frac{n!}{k!(n-k)!} p^k q^{n-k} \right) + < k >= n^2 p^2 - np^2 + np$$

and finally $\sigma^2(k) =< k^2 > - < k >^2 = np(1-p)$.

Note Title 2/21/2008

3) a) $\Delta Q = n C_p \, dT$

$$C_p = C_v + R = \frac{7}{2} R$$

$$\Delta Q = \frac{1000 \text{ grams}}{28 \frac{\text{grams}}{\text{mole}}} \times \frac{7}{2} \times 6 \times 10^{23} \frac{1}{\text{mole}} \times 1.38 \times 10^{-23} \frac{J}{{}^\circ K} \times 120\,{}^\circ K$$

$$= 124 \text{ kJ}$$

b) $\Delta E = n C_v \Delta T$

$$= \frac{1000}{28} \times \frac{5}{2} \times 6 \times 10^{23} \times 1.38 \times 10^{-23} \times 120$$

$$= 89 \text{ kJ}$$

c) $\Delta W = \Delta E - \Delta Q = -35 \text{ kJ}$

$\Delta W < 0$, work is done by the gas

d) $\Delta Q = n C_v \, dT = 89 \text{ kJ}$