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## Statistical Mechanics

### Assignment #8

Due : Monday 04/20/09 at 5pm

Reminder:

- This is the final homework. There will be a Q&A section on the last lecture of the class (Friday, 04/24), you can ask any questions about the class, final exam that day.
- The final exam is on May 1st from 2:50pm-5:40pm according to the official school calendar of the institute. For some of you who need ADAPTS services, please contact them as soon as you can.
- The final topics will be about various ensembles and quantum stat mech. It will not cover thermodynamics, statistics or interacting gas. However, basic knowledge of the thermodynamic potentials and various relations is still required. Once again, the difficulty levels are similar to your homework problems. Please go through all of these problems carefully.
- If you have problems with grades of various homeworks, tests, you need to contact/see me before Monday 5/4 at noon. After that time and date, everything course-related is finalized with the institute and it would be extremely hard for me to change any grades after that.
- The last two weeks of the class, I'll try to teach you various approximated analytical and phenomenological methods to deal with (classical) interacting gas. These topics are for your knowledge and are not covered in the final exam.
- Please leave feedback/evaluation for the class at <http://www.coursesurvey.gatech.edu/> before you leave for summer. I'd appreciate any comments/suggestions you have.

#### Problem 1

A historic failure of classical physics is its description of the electromagnetic radiation from a black body. Consider a simple model for an ideal black body consisting of a cubic cavity of side  $L$  with a small hole in one side.

- (a) Assuming the classical equipartition of energy, derive an expression for the average energy per unit volume and unit frequency range (Rayleigh-Jeans' law).
- (b) Repeat the calculation, now using quantum ideas. The obtained spectral distribution is called Planck's law. It properly explains the experimentally observed distribution.
- (c) Find the temperature dependence of the total power emitted from the hole.

#### Problem 2

Our universe is filled with black body radiation (photons) at a temperature  $T$ . Express the photon number density  $n$  analytically in terms of  $T$  and universal constants. However, a certain numerical cofactor may be left in the form of a dimensionless integral which need not be evaluated.

#### Problem 3

Calculate the density of states,  $g(\varepsilon) = dN/d\varepsilon$  for a relativistic electron gas.

Hint:  $\varepsilon = pc$ ,  $p = h/\lambda_n$  where  $\lambda_n = 2L/n$ .

**Problem 4** An  $n$ -dimensional universe.

In our three-dimensional universe, the following are well-known results from statistical mechanics and thermodynamics :

- (a) The energy density of black body radiation depends on the temperature as  $T^\alpha$ , where  $\alpha = 4$ .
- (b) In the Debye-model of a solid, the specific heat at low temperatures depends on the temperature as  $T^\beta$ , where  $\beta = 3$ .
- (c) The ratio of the specific heat at constant pressure to the specific heat at constant volume for a monoatomic ideal gas is  $\gamma = 5/3$ .

Derive the analogous results (i.e., what are  $\gamma, \alpha$  and  $\beta$ ) in the universe with  $n$  dimensions.

**Problem 5**

Calculate the Fermi energy for a one-dimensional metal with one free electron per atom and an atomic spacing of  $2.5\text{\AA}$  at  $T = 0$ .

**Problem 6**

Consider a quantum non-interacting gas with energy  $\epsilon = c|k|^\alpha$  where  $c$  and  $\alpha$  are constants. Show that the internal energy of the gas  $E$  is proportional to  $PV$ . Find the coefficient of proportionality if the gas molecules are a) fermionic, b) bosonic.