PHYS 6107 homework 5, due Monday 03/09 5pm

1. Exercise 7.9. Stat. Mech. can be used to calculate not only macroscopic thermodynamic quantities (energy, entropy, specific heat,..), but also averaged microscopic quantities. For example, the averaged relative momenta $|\vec{p}_i - \vec{p}_k|$ between particles *i* and *k* in an ideal gas can be calculated. Using the probability distribution for this relative momenta:

   $f_{ik}(p) = \langle \delta(p - |\vec{p}_i - \vec{p}_k|) \rangle$,

   calculate the mean $< |\vec{p}_i - \vec{p}_k| >$ for an ideal gas of N particles at temperature T.
2. Read section 7.5 and derive expression Eq. (7.133) for the probability density function of a system energy near the mean value <E>. (it's a Gaussian as we learned from Chapter 2 on probability theory).
3. Section 8.2: *Quantum magnetic dipoles* (In class we studied paramagnetism of a classical magnetic dipoles) : The energy of magnetic dipoles with total angular momentum $\vec{j}$ in a magnetic field H is $E = -\vec{\mu} \cdot \vec{H} = -g\mu_B Hm$, where *g*, μ_B are the gyromagnetic ratio and magneton Bohr correspondingly (they're constants), $m = -j, -(j-1), -(j-2), \ldots, (j-1), j$, is the component of $\vec{j}$ along the magnetic field direction (for j=1/2, *m* would either be −1/2 or +1/2). Write down the single particle partition function. Calculate the free energy, entropy, energy, mean total magnetic moment, magnetic susceptibility, specific heat of a system of N such dipoles.

4. *Gas of classical rotors*. The Hamiltonian of a symmetric 3D rotor has the following form:

   $$H = \frac{p^2}{2m} + \frac{p_\theta^2}{2I_1} + \frac{p_\psi^2}{2I_3} + \frac{(p_\varphi - p_\psi \cos\theta)^2}{2I_1 \sin^2\theta}$$

   p is the momentum conjugate to the position of the center of mass of the rotor. $p_{\theta,\psi,\varphi}$ are the momentums conjugate to the corresponding rotational angles θ, ψ, and φ , $\theta \in [0,\pi]$; $\psi, \varphi \in [0, 2\pi]$. A gas of N such rotors is in contact with a heat reservoir at temperature T. Calculate the partition function, free energy, entropy, heat capacity of the gas. How is the heat capacity behave at small, large temperature ?

5. Consider an adsorbent surface having N sites, each of which can adsorb one gas molecule. This surface is in contact with an ideal gas with chemical potential m (determined by the pressure p and the temperature T). Assuming that the adsorbed molecule has energy $-\varepsilon_0$ compared to one in a free state

   (a) find the grand canonical partition function and

   (b) calculate the covering ratio θ, i.e. the ratio of adsorbed molecules to adsorbing sites on the surface.

   Hint : $(1+x)^N = \sum_{k=0}^{N} x^k \frac{N!}{k!(N-k)!}$.
6. Calculate the partition function, internal energy, pressure, heat capacity of an ideal gas of N molecules with kinetic energy $H = \sum_{i=1}^{N} A|\vec{p}_i|^\alpha$, A and α are constants. Do this using canonical ensemble. (Notice how the equipartition theorem is changed for non-harmonic gas.).
7. Do problem 6 using grand-canonical ensemble.