

### PHYS 6107, Homework 4, Due Friday 02/13 at 5pm

(This is the last homework before Midterm 1 (Friday 02/20). All homeworks will be returned and solutions posted Monday 02/16)

1. A hydrogen molecule can exist in two forms: para-hydrogen (where the two nuclear spins are anti-parallel) and the ortho-hydrogen (where the spins are parallel). The ortho-hydrogen has three spin states all of the same energy  $\epsilon > 0$ , the para-hydrogen has one spin state with energy 0. A solid is made of  $N$  hydrogen molecules, each are distinguishable from others by being localized on a lattice site. The system is isolated with energy  $E = n \epsilon$ . The spins of the different molecules are non-interacting.  
Obtain an expression for the temperature and specific heat of this system?  
How does the specific heat vary with temperature at low temperature?  
What is the limiting value for the specific heat at high temperature? Explain.  
What is the probability that a given molecule is in one particular ortho-hydrogen form?
2. An idealized model of a crystal has  $N$  lattice points and  $N$  interstitial positions. Let  $e$  be the energy needed to remove an atom from a lattice site to an interstitial position and let  $n$  be the number of atoms occupying interstitial sites in equilibrium. Calculate the entropy of the system. In equilibrium at temperature  $T$ , the Helmholtz free energy is minimized. Use this fact to calculate how many such defects are there in the solid (i.e. what is  $n$ )?
3. A 1D polymer is made of  $N$  monomers linked together end-to-end. The angle between neighbouring links is either  $0^\circ$  or  $180^\circ$ . Find the entropy of such polymer where its total length is  $L=2md$  ( $d$  is the length of a link and  $m$  is an interger  $\ll N$ ). Calculate the spring constant of the polymer (notice the polymer elasticity is purely entropic in origin).
4. Same system as problem 1. The system is not isolated but is in equilibrium with reservoir at constant temperature  $T$ . Calculate the total energy, specific heat using canonical ensemble.
5. Redo exercise 7.2. Compare the result to problem 7 of HW3.
6. A zipper has  $N$  links; each link has a state in which it is closed with energy 0 and a state in which it is open with energy  $e$ . We require that the zipper only unzip from one side (say from left) and that the link can only open if all links to the left of it (1,2,... $n-1$ ) are already open. (This model is sometimes used for DNA zipping.)
  - a. Find the partition function.
  - b. Find the average number of open links  $\langle n \rangle$  and show that for low temperatures  $k_B T \ll e$ ,  $\langle n \rangle$  is independent of  $N$ .