PHYS 4251 Midterm 1

Problem 1 (5pts): In Craps dice game, player throws two dices. If the outcome is 7 or 11, he wins. If the outcome is 2, 3 or 12, he loses. What is the probability that he'll win after a throw? What is the probability that he'll lose after a throw?

Problem 2 (2pts): Newton second law states that the acceleration of a particle is proportional to the total force acting on it: $\vec{a}=\vec{F}/m$. In class you learned that, because water is a viscous media (made of a large number of molecules), the velocity of a particle is proportional to the external force acting on it: $\vec{v}=\vec{F}/\zeta$. Does this mean Newton law doesn't apply to motion of particles in water? Explain your answer.

Problem 3 (5pts): The uniform distribution function P(x) is defined as P(x) = const if $0<x<a$, and P(x) = 0 if $x<0$ or $x>a$.

What is the value of the const? Calculate the mean, variance of this distribution function.

Problem 4: *Centrifugation.* A glass tube contains a solution of particles with atomic mass *m*, inverse mobility ζ. The tube is then rotated with angular velocity *w* about an axis perpendicular to the tube axis. The centripetal force acting on a solute particle is $f=-mrw^2$, where *r* is the distance from the particle to the axis of rotation. This force obviously comes from the surrounding fluid as the solute particle drifts outward. We're going to calculate the equilibrium solute concentration in the tube using two different methods:

a) (4pts) Method 1: in equilibrium, the solute concentration profile obeys Boltzmann distribution. Assuming the temperature is uniform throughout the tube, T=300ºK, the variation in concentration is only due to the variation in the potential energy. As a solute particle move a distance *dr* away from the axis of rotation, the change in the potential energy is $dU=f\,dr$. Calculate the total change in potential energy as a solute particle moves from 0 to a distance *r*. Use this result to write down the concentration $c(r)$. (the concentration at r = 0 is a constant $c(0) = c_0$).

b) (Optional question, 3pts) Method 2: write down the expression for the total current of the solute (sum of diffusion current and a drift current due to the viscous force). In equilibrium, this total current is zero. Calculate the equilibrium concentration $c(r)$ as function of the distance *r* from the rotation axis.

c) (4pts) The solute particle is a protein with molecular mass of 30 000Da. You want to obtain a 10 times difference between the protein concentrations at the top (r = 0 cm) and at the bottom (r = 10cm) of the tube. How fast do you have to rotate the tube?

Note: the maximum score for this test is 23/20 or 115%.

Solution

Problem 1:
He wins if the dices are "1 and 6" or "2 and 5" or "3 and 4" or "4 and 3" or "5 and 2" or "6 and 1" or "5 and 6" or "6 and 5". In other words, the probability that he wins is

P(1) P(6) + P(2) P(5) + P(3) P(4) + P(4) P(3) + P(5) P(2) + P(6) P(1) + P(5) P(6) + P(6) P(5) = 8/36 = 22%

Similarly, the probability that he loses is

P(1) P(1) + P(1) P(2) + P(2) P(1) + P(6) P(6) = 4/36 = 11%

Problem 2:

Newton law still applies. The acceleration of a particle in water is zero (constant velocity). This means the *total* force acting on it is zero. The total force is the sum of the *external* force and viscous force. The later equals $-\zeta v$.

$$F_{total}=F_{ext}+F_{vis}=\vec{F}-\zeta v_{drift}=0 \quad .$$

Problem 3:

From the normalization condition, $\int_0^a P(x)\,dx=1$, we obtain const = 1/a.

The mean is $\langle x\rangle=\int_0^a x\,P(x)\,dx=\frac{1}{a}\int_0^a x\,dx=\frac{a}{2}$.

The variance is $\left\langle x^2-\langle x\rangle^2\right\rangle=\int_0^a (x^2-\langle x\rangle^2)P(x)\,dx=\int_0^a (x^2-\frac{a^2}{4})\frac{1}{a}\,dx=\frac{a^2}{3}-\frac{a^2}{4}=\frac{a^2}{12}$.

Problem 4:

a) The change in the potential energy is $U=\int_0^r dU=-\int_0^r mrw^2\,dr=-\frac{mr^2w^2}{2}$. Boltzmann distribution states that the concentration is proportional to inverse exponential of the energy. Since the temperature is constant throughout the tube, the concentration is proportional to the inverse exponential of the potential energy:

$$c(r)=c(0)e^{-U/k_BT}=c_0 e^{mr^2w^2/2k_BT} \quad .$$

b) The total current is

$$j_{total}=j_{diff}+j_{drift}=-D\frac{dc}{dr}-c\frac{f}{\zeta}=-D\frac{dc}{dr}+c\frac{mrw^2}{k_BT/D}=-D\left(\frac{dc}{dr}-c\frac{m\,r\,w^2}{k_BT}\right) \quad .$$

In equilibrium, the total current is zero, we get

$$\frac{dc}{dr} = c\frac{m\,r\,w^2}{k_B T}$$

The solution to this equation is $$c(r) = c(0)e^{\frac{m r^2 w^2}{2 k_B T}} \quad .$$

c) We want to have c (r = 10cm) / c (r = 0cm) = 10. Thus $\exp\left(\frac{m r^2 w^2}{2 k_B T}\right) = 10$.

Substitute the values, $k_B = 1.38 \times 10^{-23}$ J/K, T = 300 K, m = 30 000 × 1.66 × 10^{-27} kg, we get

$$w = \sqrt{\frac{(2\ln 10)\,1.38\times 10^{-23}\times 300}{30000\times 1.66\times 10^{-27}(10\times 10^{-2})^2}} = 196\,rad/s = 1800\,rpm \quad .$$