PROBLEMS

6.1 *Tall tale*

The mythical lumberjack Paul Bunyan usually cut down trees, but on one occasion he attempted to diversify and run his own sawmill. As the historians tell it, "Instead of turning out lumber the mill began to take in piles of sawdust and turn it back into logs. They soon found out the trouble: A technician had connected everything up backwards."

Can we reject this story on the basis of the Second Law?

6.2 *Entropy change upon equilibration*

Consider two boxes of ideal gas. The boxes are thermally isolated from the world and, initially, from each other as well. Each box holds N molecules in volume V. Box *1* starts with temperature $T_{i,1}$, whereas box *2* starts with $T_{i,2}$. (The subscript "i" means "initial," and "f" will mean "final.") So the initial total energies are $E_{i,1} = N\frac{3}{2}k_B T_{i,1}$ and $E_{i,2} = N\frac{3}{2}k_B T_{i,2}$.

Now we put the boxes into thermal contact with each other but still isolated from the rest of the world. We know they'll eventually come to the same temperature, as argued in Equation 6.10.

a. What is this temperature?

b. Show that the change of total entropy S_{tot} is then

$$k_B \frac{3}{2} N \ln \frac{(T_{i,1} + T_{i,2})^2}{4 T_{i,1} T_{i,2}}.$$

c. Show that this change is always ≥ 0. [*Hint:* Let $X = \frac{T_{i,1}}{T_{i,2}}$ and express the change of entropy in terms of X. Plot the resulting function of X.]

d. Under a special circumstance, the change in S_{tot} will be zero: When? Why?

6.3 *Bobble Bird*

The Bobble Bird toy dips its beak into a cup of water, rocks back until the water has evaporated, then dips forward and repeats the cycle. All you need to know about the internal mechanism is that after each cycle, it returns to its original state: There is no spring winding down and no internal fuel getting consumed. You could even attach a little ratchet to the toy and extract a little mechanical work from it, maybe lifting a small weight.

a. Where does the energy to do this work come from?

b. Your answer in (a) may at first seem to contradict the Second Law. Explain why it does not. [*Hint:* What system discussed in Chapter 1 does this device resemble?]

6.4 *Efficient energy storage*

Section 6.5.3 discussed an energy-transduction machine. We can see some similar lessons from a simpler system, an energy-*storage* device. Any such device in the cellular world will inevitably lose energy, as a result of viscous drag, so we imagine pushing a ball through a viscous fluid. We push with constant external force f; as the ball moves, it compresses a spring (Figure 6.11). According to the Hooke relation, the

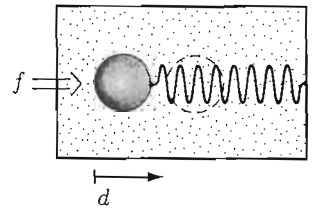

Figure 6.11: A simple energy-storage device. A tank filled with a viscous fluid contains an elastic element (spring) and a bead, whose motion is opposed by viscous drag.

spring resists compression with an elastic force $f = kd$, where k is the spring constant.[8] When this force balances the external force, the ball stops moving, at $d = f/k$.

Throughout the process, the applied force was fixed, so by this point we've done work $fd = f^2/k$. But integrating the Hooke relation shows that our spring has stored only $\int_0^d f(x)\mathrm{d}x = \frac{1}{2}kd^2$, or $\frac{1}{2}f^2/k$. The rest of the work we did went to generate heat. Indeed, at every position x along the way from 0 to d, some of the applied force compresses the spring while the rest goes to overcome viscous friction.

Nor can we get back all the stored energy, $\frac{1}{2}f^2/k$, because we lose even more to friction as the spring relaxes. Suppose that we suddenly reduce the external force to a value f_1 that is smaller than f.

a. Find how far the ball moves and how much work it does against the external force. We'll call the latter quantity the "useful work" recovered from the storage device.
b. For what constant value of f_1 will the useful work be maximal? Show that even with this optimal choice, the useful work output is only half of what was stored in the spring, or $\frac{1}{4}f^2/k$.
c. How could we make this process more efficient? [*Hint:* Keep in mind Idea 6.20.]

6.5 *Atomic polarization*

Suppose that we have a lot of noninteracting atoms (a gas) in an external magnetic field. You may take as given the fact that each atom can be in one of two states, whose energies differ by an amount $\Delta E = 2\mu B$, depending on the strength of the magnetic field B. Here μ is some positive constant, and B is also positive. Each atom's magnetization is taken to be +1 if it's in the lower energy state or −1 if it's in the higher state.

a. Find the *average* magnetization of the entire sample as a function of the applied magnetic field B. [*Remark:* Your answer can be expressed in terms of ΔE by using a hyperbolic trigonometric function; if you know these, then write it this way.]
b. Discuss how your solution behaves when $B \to \infty$ and when $B \to 0$, and why your results make sense.

6.6 *Polymer mesh*

D. Discher studied the mechanical character of the red blood cell cytoskeleton, a polymer network attached to its inner membrane. Discher attached a bead of diameter

[8] Another Hooke relation appeared in Chapter 5, where the force resisting a shear deformation was proportional to the size of the deformation (Equation 5.14 on page 172).

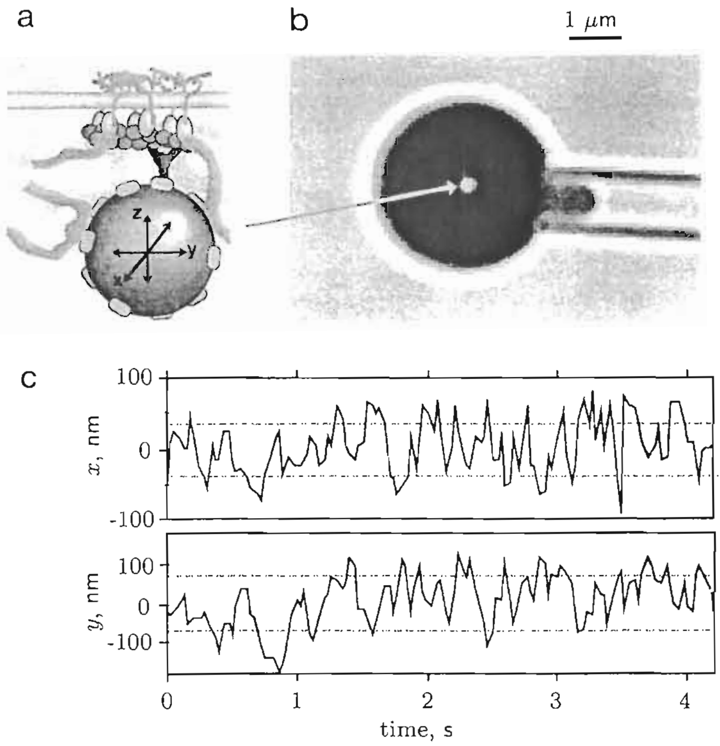

Figure 6.12: (Schematic; optical micrograph; experimental data.) (a) Attachment of a single fluorescent nanoparticle to actin in the red blood cell cortex. (b) The red cell, with attached particle, is immobilized by partially sucking it into a micropipette (*right*) of diameter 1 μm. (c) Tracking of the thermal motion of the nanoparticle gives information about the elastic properties of the cortex. [Digital image kindly supplied by D. Discher; see Discher, 2000.]

40 nm to this network (Figure 6.12a). The network acts as a spring, constraining the free motion of the bead. He then asked, "What is the stiffness (spring constant) of this spring?"

In the macroworld, we'd answer this question by applying a known force to the bead, measuring the displacement Δx in the x direction, and using $f = k\Delta x$. But it's not easy to apply a known force to such a tiny object. Instead, Discher just passively observed the thermal motion of the bead (Figure 6.12c). He found the bead's root-mean-square deviation from its equilibrium position, at room temperature, to be $\sqrt{\langle(\Delta x)^2\rangle} = 35$ nm; from this, he computed the spring constant k. What value did he find?

6.7 *Inner ear*

A. J. Hudspeth and coauthors found a surprising phenomenon while studying signal transduction by the inner ear. Figure 6.13a shows a bundle of stiff fibers (called stereocilia) projecting from a sensory cell. The fibers sway when the surrounding inner-ear