Problems

1.1

Ans:
a. No contradiction. The heat removed from the room, plus the electrical energy input, all gets exhausted to the outside world.
b. Only if it's hotter in your room than it is in the outside world! In that case, you could install a heat engine in your window and use it to generate electricity. Formally, transferring heat from your room decreases the temperature difference, which is a form of order, and may proceed spontaneously.

1.2

Ans:
a. First note that Earth's gravity pulls on a kilogram of water with a force of $1\,\mathsf{kg} \times g$. We were told that this force equals $2.2\,\mathsf{pound}$. The mass of $25.5\,\mathsf{pound}$ is $(25.5\,\mathsf{pound}/g)$. Using the value for the mechanical equivalent of heat given in Equation 1.2, we find the power needed to raise the temperature of water from $20°\mathsf{C}$ to $100°\mathsf{C}$:

$$\frac{25.5\,\mathsf{pound}/g}{(2.2\,\mathsf{pound})/(1\,\mathsf{kg} \times g)} \frac{10^3\,\mathsf{g}}{\mathsf{kg}} \times (80°\mathsf{C}) \times \frac{\mathsf{cal}}{1\,\mathsf{g}\,°\mathsf{C}} \times \frac{1\,\mathsf{J}}{0.24\,\mathsf{cal}} \times \frac{1}{9000\,\mathsf{s}} = 430\,\mathsf{W}.$$

b. This time we are to *find* the mechanical equivalent of heat. It's given by (work done)/(temperature change×mass):

$$\left((770\,\mathsf{foot\ pound})\frac{12\,\mathsf{inch}}{1\,\mathsf{foot}}\frac{2.54\,\mathsf{cm}}{1\,\mathsf{inch}}\frac{1\,\mathsf{m}}{100\,\mathsf{cm}}\right)\frac{1}{0.56°\mathsf{C}}\left(\frac{1\,\mathsf{pound}}{g}\right)^{-1}$$

$$= 4.11\frac{\mathsf{J}}{°\mathsf{C}\,\mathsf{g}}.$$

That's quite close to the modern value.

1.3

Ans:
a. The energy yield per liter of O_2 consumed and ratio of gas exchanges can be obtained by dividing appropriate columns from the table in the problem. The results for different foods are

Food	kcal/L(O_2)	L(CO_2)/L(O_2)
Carbohydrate	5.06	1.00
Fat	4.74	0.71
Protein	4.25	0.80
Alcohol	4.86	0.66

listed in the table below From the table we see that the energy yielded per liter of O_2 consumed is roughly constant, about 5 kcal per liter of O_2. On the other hand, the CO_2/O_2 ratio varies between approximately 2/3 and 1 depending on a particular food used.
b. The basal metabolic rate (BMR) is the energy per time liberated by metabolizing food. Thus,

$$\mathrm{BMR} \approx 5\,\frac{\mathsf{kcal}}{\mathsf{L}(\mathrm{O}_2)} \times 16\,\frac{\mathsf{L}(\mathrm{O}_2)}{\mathrm{hour}} \approx 80\,\frac{\mathsf{kcal}}{\mathrm{hour}} \approx 1900\,\frac{\mathsf{kcal}}{\mathrm{day}}.$$

c. In order to convert the BMR from kcal/hour to watts we need to remember that 1 W= 1 J/s, as well as the kcal to J conversion:

$$\mathrm{BMR} \approx 80\,\frac{\mathsf{kcal}}{\mathrm{hour}} \times \frac{1\mathrm{hour}}{3600\ \mathsf{s}} \times \frac{4.18\ \mathsf{kJ}}{\mathsf{kcal}} \approx 90\ \mathsf{W}.$$

d. We use the CO_2 output rate together with the O_2 consumption given in (b) to compute the ratio of 0.84. This number is clearly higher than the average of the foods surveyed, indicating that if a person's intake includes all the given foods then a significant portion of his/her diet contains carbohydrates.
e. To compute efficiency we need to compare the work done with the excess energy consumed. The work done in a day is a product of the power output and the time:

$$W = 50\,\frac{\mathsf{J}}{\mathsf{s}} \times 10\ \mathrm{hour} \times \frac{1\ \mathsf{kcal}}{4180\ \mathsf{J}} \approx 430\ \mathsf{kcal}.$$

Excess energy consumed is the difference between the given energy intake and the BMR. This difference equals 1600 kcal. The ratio of work done and excess energy consumed gives the efficiency of about 27%.

1.4

Ans:
Equating the expressions for total absorbed and reradiated energies one can obtain a condition for water-based life:

$$\begin{aligned} \alpha\sigma T^4 \times 4\pi R^2 &= \pi R^2 \times \alpha I_e (d_e/d)^2 \\ 4\sigma T^4 &= I_e (d_e/d)^2. \end{aligned}$$

a. To compute the average temperature on Earth we set $d = d_e$ and use the given value for incident flux, I_e and the Stefan-Boltzmann constant, σ:

$$\begin{aligned} T^4 &= \frac{1.4\ \mathsf{kW/m^2}}{4 \times 5.7 \times 10^{-8}\ \mathsf{W/(m^2 \cdot K^4)}} \\ T &= 280\ \mathsf{K}. \end{aligned}$$

Luckily we don't need to know the value of α; it dropped out. (So does R.) The obtained value is remarkably close to the actual value of 289 K.

b. Computing the orbit of a planet given a mean temperature is just a reverse of the operation in part (a). So for mean temperature of 273 K we obtain:

$$\begin{aligned}(d/d_\mathrm{e})^2 &= \frac{1.4\ \mathsf{kW/m^2}}{4 \times 5.7 \times 10^{-8}, \mathsf{W/(m^2 \cdot K^4)} \times (273\ \mathsf{K})^4} \\ d &= 1.05 d_\mathrm{e}.\end{aligned}$$

c. Similarly, at T = 373 K:

$$\begin{aligned}(d/d_\mathrm{e})^2 &= \frac{1.4\ \mathsf{kW/m^2}}{4 \times 5.7 \times 10^{-8}\, \mathsf{W/(m^2 \cdot K^4)} \times (373\ \mathsf{K})^4} \\ d &= 0.6 d_\mathrm{e}.\end{aligned}$$

d. We look up the data for the planets' orbits (in millions of km): Mercury 58, Venus 108, Earth 150, Mars 228 and Jupiter 778. We then see that based on this oversimplified criterion, the candidates for water-based life must be between 85 and 158 million km from the Sun. This leaves Earth and Venus as the only candidates.

We can't take this result completely seriously. For one thing, geothermal energy can also add to a planet's surface temperature, perhaps at one time putting Mars over the threshold. Also, we can let the poles freeze, as they do on Earth, and still have liquid water on the Equator. And indeed, NASA announced in June 2000 that they found evidence of liquid water long ago on Mars. On the other hand, the real temperature of Venus is considerably higher than the estimate found in the simple model considered here.

In any case, we do see why we don't expect any water-based life on Mercury or Neptune.

Homework 1, problem 5:

One mole of water weighs 18grams. The density of water is 1000grams/litter=10^6g/m^3. So the volume of one mole of water is,

$$V_{mole} = \frac{18\,g}{10^6\,g/m^3} = \frac{18}{10^6}\,m^3 \quad .$$

There are N_{av}=6.022×10^{23} molecules in one mole of water. Therefore, the volume of one water molecule is:

$$V_{molecule} = \frac{V_{mole}}{N_{av}} = \frac{18}{10^6 \times 6.022 \times 10^{23}}\,m^3 = \frac{18}{6.022 \times 10^{29}}\,m^3 \quad .$$

If we approximate the water molecule as a cube with size *a*, then $V_{molecule} = a^3$. Thus the size of a water molecule is

$$a = \sqrt[3]{V_{molecule}} = \sqrt[3]{\frac{18}{6.022 \times 10^{29}}}\,m = 0.31 \times 10^{-9}\,m = 0.31\,nm \quad .$$

Note: This number is actually the average distance between neighboring water molecules. The molecule itself is about 0.15nm apart between H atoms.