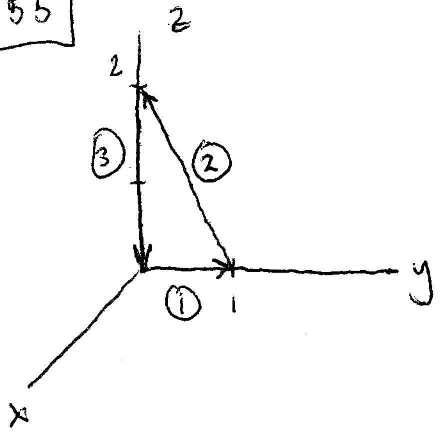
1.55



$$\vec{v} = 6\hat{x} + yz^2\hat{y} + (3y+z)\hat{z}$$

① $x = z = 0$, $y: 0 \to 1$, $d\vec{l} = dy\,\hat{y}$

$$\vec{v}\cdot d\vec{l} = yz^2\,dy \equiv \boxed{0}$$

② $x = 0$, $y: 1 \to 0$, $z = -2y + 2 \Rightarrow dz = -2dy$

$$d\vec{l} = dy\,\hat{y} + dz\,\hat{z} = dy\,\hat{y} - 2\,dy\,\hat{z}, \quad z^2 = 4y^2 - 8y + 4$$

$$\vec{v}\cdot d\vec{l} = yz^2\,dy - 2(3y+z)\,dy = (4y^3 - 8y^2 + 2y - 4)\,dy$$

$$\int_1^0 (4y^3 - 8y^2 + 2y - 4)\,dy = -1 + \frac{8}{3} - 1 + 4 = \boxed{\frac{14}{3}}$$

③ $x = y = 0$ $\quad d\vec{l} = dz\,\hat{z}$ $\quad \vec{v}\cdot d\vec{l} = (3y+z)\,dz = z\,dz$

$z: 2 \to 0$ $\qquad \displaystyle\int_2^0 z\,dz = \frac{z^2}{2}\Big|_2^0 = \boxed{-2}$

$$\oint \vec{v}\cdot d\vec{l} = \int_{①} + \int_{②} + \int_{③} = 0 + \frac{14}{3} - 2 = \boxed{\frac{8}{3}}$$

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 6 & yz^2 & 3y+z \end{vmatrix} = (3 - 2yz)\,\hat{x} \qquad \int(\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \int_0^1\left(\int_0^{2-2y}(3-2yz)\,dz\right)dy$$

$d\vec{a} = dy\,dz\,\hat{x}$ $\qquad \displaystyle\int(\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \int_0^1 (3z - yz^2)\Big|_0^{2-2y} dy = \int_0^1 \left(6 - 6y - (4y^3 - 8y^2 + 4y)\right)dy$

$$= \int_0^1 (-4y^3 + 8y^2 - 10y + 6)\,dy = -1 + \frac{8}{3} - 5 + 6 = \boxed{\frac{8}{3}}$$