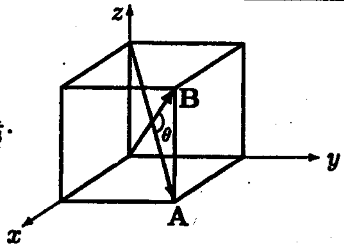
Problem 1.3



$\mathbf{A} = +1\,\hat{\mathbf{x}} + 1\,\hat{\mathbf{y}} - 1\,\hat{\mathbf{z}};\ A = \sqrt{3};\ \mathbf{B} = 1\,\hat{\mathbf{x}} + 1\,\hat{\mathbf{y}} + 1\,\hat{\mathbf{z}};\ B = \sqrt{3}.$

$\mathbf{A}\cdot\mathbf{B} = +1 + 1 - 1 = 1 = AB\cos\theta = \sqrt{3}\sqrt{3}\cos\theta \Rightarrow \cos\theta = \frac{1}{3}.$

$\boxed{\theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5288^\circ}$

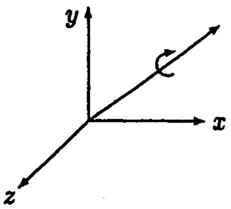
Problem 1.7

$\boldsymbol{\mathscr{r}} = (4\,\hat{\mathbf{x}} + 6\,\hat{\mathbf{y}} + 8\,\hat{\mathbf{z}}) - (2\,\hat{\mathbf{x}} + 8\,\hat{\mathbf{y}} + 7\,\hat{\mathbf{z}}) = \boxed{2\,\hat{\mathbf{x}} - 2\,\hat{\mathbf{y}} + \hat{\mathbf{z}}}$

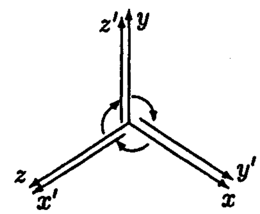
$\mathscr{r} = \sqrt{4+4+1} = \boxed{3}$

$\hat{\boldsymbol{\mathscr{r}}} = \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}} = \boxed{\frac{2}{3}\hat{\mathbf{x}} - \frac{2}{3}\hat{\mathbf{y}} + \frac{1}{3}\hat{\mathbf{z}}}$

Problem 1.9



Looking down the axis:



A 120° rotation carries the z axis into the y $(= \bar{z})$ axis, y into x $(= \bar{y})$, and x into z $(= \bar{x})$. So $\overline{A}_x = A_z$, $\overline{A}_y = A_x$, $\overline{A}_z = A_y$.

$$\boxed{R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}$$

Problem 1.12

(a) $\boldsymbol{\nabla} h = 10[(2y - 6x - 18)\,\hat{\mathbf{x}} + (2x - 8y + 28)\,\hat{\mathbf{y}}]$. $\boldsymbol{\nabla} h = 0$ at summit, so

$$\left.\begin{array}{l} 2y - 6x - 18 = 0 \\ 2x - 8y + 28 = 0 \Longrightarrow 6x - 24y + 84 = 0 \end{array}\right\} 2y - 18 - 24y + 84 = 0.$$

$22y = 66 \Longrightarrow y = 3 \Longrightarrow 2x - 24 + 28 = 0 \Longrightarrow x = -2.$

Top is $\boxed{\text{3 miles north, 2 miles west, of South Hadley.}}$

(b) Putting in $x = -2$, $y = 3$:

$h = 10(-12 - 12 - 36 + 36 + 84 + 12) = \boxed{720 \text{ ft.}}$

(c) Putting in $x = 1$, $y = 1$: $\boldsymbol{\nabla} h = 10[(2 - 6 - 18)\,\hat{\mathbf{x}} + (2 - 8 + 28)\,\hat{\mathbf{y}}] = 10(-22\,\hat{\mathbf{x}} + 22\,\hat{\mathbf{y}}) = 220(-\hat{\mathbf{x}} + \hat{\mathbf{y}})$.

$|\boldsymbol{\nabla} h| = 220\sqrt{2} \approx \boxed{311 \text{ ft/mile}}$; direction: $\boxed{\text{northwest.}}$

Problem 1.15

(a) $\boldsymbol{\nabla}\cdot\mathbf{v}_a = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(3xz^2) + \frac{\partial}{\partial z}(-2xz) = 2x + 0 - 2x = 0.$

(b) $\boldsymbol{\nabla}\cdot\mathbf{v}_b = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(3xz) = y + 2z + 3x.$

(c) $\boldsymbol{\nabla}\cdot\mathbf{v}_c = \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(2xy + z^2) + \frac{\partial}{\partial z}(2yz) = 0 + (2x) + (2y) = 2(x + y).$