$\mathbf{E}\cdot d\mathbf{l} = 2kyz\,dz = 2ky_0 z\,dz.$
$\int_{III}\mathbf{E}\cdot d\mathbf{l} = 2y_0 k\int_0^{z_0} z\,dz = ky_0 z_0^2.$

$$V(x_0,y_0,z_0) = -\int_0^{(x_0,y_0,z_0)}\mathbf{E}\cdot d\mathbf{l} = -k(x_0y_0^2 + y_0z_0^2),\ \text{or}\ \boxed{V(x,y,z) = -k(xy^2+yz^2).}$$

Check: $-\nabla V = k\left[\frac{\partial}{\partial x}(xy^2+yz^2)\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}(xy^2+yz^2)\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}(xy^2+yz^2)\,\hat{\mathbf{z}}\right] = k[y^2\,\hat{\mathbf{x}} + (2xy+z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}}] = \mathbf{E}.$ ✓

Problem 2.21

$$V(r) = -\int_\infty^r \mathbf{E}\cdot d\mathbf{l}.\quad \begin{cases}\text{Outside the sphere } (r>R): & \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.\\ \text{Inside the sphere } (r<R): & \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{\mathbf{r}}.\end{cases}$$

So for $r>R$: $V(r) = -\int_\infty^r\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\bar r^2}\right)d\bar r = \frac{1}{4\pi\epsilon_0}q\left(\frac{1}{\bar r}\right)\Big|_\infty^r = \boxed{\frac{q}{4\pi\epsilon_0}\frac{1}{r},}$

and for $r<R$: $V(r) = -\int_\infty^R\left(\frac{1}{4\pi\epsilon_0}\frac{q}{\bar r^2}\right)d\bar r - \int_R^r\left(\frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\bar r\right)d\bar r = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{R^3}\left(\frac{r^2-R^2}{2}\right)\right]$

$$= \boxed{\frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(3-\frac{r^2}{R^2}\right).}$$

HW#5

When $r>R$, $\nabla V = \frac{q}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\hat{\mathbf{r}} = -\frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{\mathbf{r}}$, so $\mathbf{E} = -\nabla V = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{\mathbf{r}}.$ ✓

When $r<R$, $\nabla V = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\frac{\partial}{\partial r}\left(3-\frac{r^2}{R^2}\right)\hat{\mathbf{r}} = \frac{q}{4\pi\epsilon_0}\frac{1}{2R}\left(-\frac{2r}{R^2}\right)\hat{\mathbf{r}} = -\frac{q}{4\pi\epsilon_0}\frac{r}{R^3}\hat{\mathbf{r}}$; so $\mathbf{E} = -\nabla V = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{\mathbf{r}}.$ ✓

Problem 2.22

$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{s}\hat{\mathbf{s}}$ (Prob. 2.13). In this case we cannot set the reference point at ∞, since the charge itself extends to ∞. Let's set it at $s=a$. Then

$$V(s) = -\int_a^s\left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{\bar s}\right)d\bar s = \boxed{-\frac{1}{4\pi\epsilon_0}2\lambda\ln\left(\frac{s}{a}\right).}$$

(In this form it is clear why $a=\infty$ would be no good—likewise the other "natural" point, $a=0$.)

$\nabla V = -\frac{1}{4\pi\epsilon_0}2\lambda\frac{\partial}{\partial s}\left(\ln\left(\frac{s}{a}\right)\right)\hat{\mathbf{s}} = -\frac{1}{4\pi\epsilon_0}2\lambda\frac{1}{s}\hat{\mathbf{s}} = -\mathbf{E}.$ ✓

Problem 2.23

$$V(0) = -\int_\infty^0\mathbf{E}\cdot d\mathbf{l} = -\int_\infty^b\left(\frac{k}{\epsilon_0}\frac{(b-a)}{r^2}\right)dr - \int_b^a\left(\frac{k}{\epsilon_0}\frac{(r-a)}{r^2}\right)dr - \int_a^0(0)dr = \frac{k}{\epsilon_0}\frac{(b-a)}{b} - \frac{k}{\epsilon_0}\left(\ln\left(\frac{a}{b}\right) + a\left(\frac{1}{a}-\frac{1}{b}\right)\right)$$

$$= \frac{k}{\epsilon_0}\left\{1 - \frac{a}{b} - \ln\left(\frac{a}{b}\right) - 1 + \frac{a}{b}\right\} = \boxed{\frac{k}{\epsilon_0}\ln\left(\frac{b}{a}\right).}$$

Problem 2.24

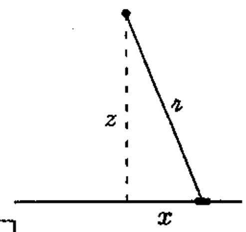
Using Eq. 2.22 and the fields from Prob. 2.16:

$$V(b) - V(0) = -\int_0^b\mathbf{E}\cdot d\mathbf{l} = -\int_0^a\mathbf{E}\cdot d\mathbf{l} - \int_a^b\mathbf{E}\cdot d\mathbf{l} = -\frac{\rho}{2\epsilon_0}\int_0^a s\,ds - \frac{\rho a^2}{2\epsilon_0}\int_a^b\frac{1}{s}ds$$

$$= -\left(\frac{\rho}{2\epsilon_0}\right)\frac{s^2}{2}\Big|_0^a + \frac{\rho a^2}{2\epsilon_0}\ln s\Big|_a^b = \boxed{-\frac{\rho a^2}{4\epsilon_0}\left(1+2\ln\left(\frac{b}{a}\right)\right).}$$

Problem 2.25

(a) $\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{2q}{\sqrt{z^2+\left(\frac{d}{2}\right)^2}}.}$



(b) $V = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\lambda\,dx}{\sqrt{z^2+x^2}} = \frac{\lambda}{4\pi\epsilon_0}\ln(x+\sqrt{z^2+x^2})\Big|_{-L}^{L}$

$$= \boxed{\frac{\lambda}{4\pi\epsilon_0}\ln\left[\frac{L+\sqrt{z^2+L^2}}{-L+\sqrt{z^2+L^2}}\right]} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{L+\sqrt{z^2+L^2}}{z}\right).$$

(c) $V = \frac{1}{4\pi\epsilon_0}\int_0^R\frac{\sigma\,2\pi r\,dr}{\sqrt{r^2+z^2}} = \frac{1}{4\pi\epsilon_0}2\pi\sigma\left(\sqrt{r^2+z^2}\right)\Big|_0^R = \boxed{\frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2}-z\right).}$

In each case, by symmetry $\frac{\partial V}{\partial y} = \frac{\partial V}{\partial x} = 0$. $\therefore \mathbf{E} = -\frac{\partial V}{\partial z}\hat{\mathbf{z}}$.

(a) $\mathbf{E} = -\frac{1}{4\pi\epsilon_0}2q\left(-\frac{1}{2}\right)\frac{2z}{\left(z^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\hat{\mathbf{z}} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{2qz}{\left(z^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}\hat{\mathbf{z}}}$ (agrees with Prob. 2.2a).

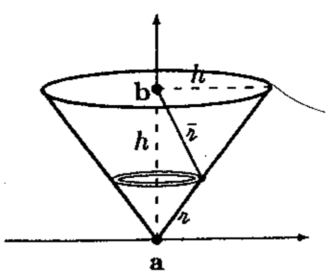
(b) $\mathbf{E} = -\frac{\lambda}{4\pi\epsilon_0}\left\{\frac{1}{(L+\sqrt{z^2+L^2})}\frac{1}{2}\frac{1}{\sqrt{z^2+L^2}}2z - \frac{1}{(-L+\sqrt{z^2+L^2})}\frac{1}{2}\frac{1}{\sqrt{z^2+L^2}}2z\right\}\hat{\mathbf{z}}$

$= -\frac{\lambda}{4\pi\epsilon_0}\frac{z}{\sqrt{z^2+L^2}}\left\{\frac{-L+\sqrt{z^2+L^2}-L-\sqrt{z^2+L^2}}{(z^2+L^2)-L^2}\right\}\hat{\mathbf{z}} = \boxed{\frac{2L\lambda}{4\pi\epsilon_0}\frac{1}{z\sqrt{z^2+L^2}}\hat{\mathbf{z}}}$ (agrees with Ex. 2.1).

(c) $\mathbf{E} = -\frac{\sigma}{2\epsilon_0}\left\{\frac{1}{2}\frac{1}{\sqrt{R^2+z^2}}2z - 1\right\}\hat{\mathbf{z}} = \boxed{\frac{\sigma}{2\epsilon_0}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]\hat{\mathbf{z}}}$ (agrees with Prob. 2.6).

If the right-hand charge in (a) is $-q$, then $\boxed{V=0}$, which, naively, suggests $\mathbf{E} = -\nabla V = 0$, in contradiction with the answer to Prob. 2.2b. The point is that we only know V *on the z axis*, and from this we cannot hope to compute $E_x = -\frac{\partial V}{\partial x}$ or $E_y = -\frac{\partial V}{\partial y}$. That was OK in part (a), because we knew from symmetry that $E_x = E_y = 0$. But now $\mathbf{E}$ points in the x direction, so knowing V on the z axis is insufficient to determine $\mathbf{E}$.

Problem 2.26



$$V(\mathbf{a}) = \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{\mathscr{r}}\right)d\mathscr{r} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{1}{\sqrt{2}}(\sqrt{2}h) = \frac{\sigma h}{2\epsilon_0}.$$

(where $r = \mathscr{r}/\sqrt{2}$)

$$V(\mathbf{b}) = \frac{1}{4\pi\epsilon_0}\int_0^{\sqrt{2}h}\left(\frac{\sigma 2\pi r}{\bar{\mathscr{r}}}\right)d\mathscr{r}, \quad \text{where } \bar{\mathscr{r}} = \sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}.$$

$$= \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{1}{\sqrt{2}}\int_0^{\sqrt{2}h}\frac{\mathscr{r}}{\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}}d\mathscr{r}$$

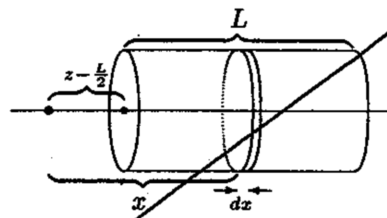
$$= \frac{\sigma}{2\sqrt{2}\epsilon_0}\left[\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}} + \frac{h}{\sqrt{2}}\ln(2\sqrt{h^2+\mathscr{r}^2-\sqrt{2}h\mathscr{r}}+2\mathscr{r}-\sqrt{2}h)\right]_0^{\sqrt{2}h}$$

$$= \frac{\sigma}{2\sqrt{2}\epsilon_0}\left[h+\frac{h}{\sqrt{2}}\ln(2h+2\sqrt{2}h-\sqrt{2}h) - h - \frac{h}{\sqrt{2}}\ln(2h-\sqrt{2}h)\right] = \frac{\sigma}{2\sqrt{2}\epsilon_0}\frac{h}{\sqrt{2}}\left[\ln(2h+\sqrt{2}h)-\ln(2h-\sqrt{2}h)\right]$$

$$= \frac{\sigma h}{4\epsilon_0}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right) = \frac{\sigma h}{4\epsilon_0}\ln\left(\frac{(2+\sqrt{2})^2}{2}\right) = \frac{\sigma h}{2\epsilon_0}\ln(1+\sqrt{2}).$$

$$\therefore \boxed{V(\mathbf{a})-V(\mathbf{b}) = \frac{\sigma h}{2\epsilon_0}\left[1-\ln(1+\sqrt{2})\right].}$$

Problem 2.27



Cut the cylinder into slabs, as shown in the figure, and use result of Prob. 2.25c, with $z \to x$ and $\sigma \to \rho\, dx$:

$$V = \frac{\rho}{2\epsilon_0}\int_{z-L/2}^{z+L/2}\left(\sqrt{R^2+x^2}-x\right)dx$$

$$= \frac{\rho}{2\epsilon_0}\frac{1}{2}\left[x\sqrt{R^2+x^2}+R^2\ln(x+\sqrt{R^2+x^2})-x^2\right]\Big|_{z-L/2}^{z+L/2}$$

$$= \boxed{\frac{\rho}{4\epsilon_0}\left\{\left(z+\tfrac{L}{2}\right)\sqrt{R^2+\left(z+\tfrac{L}{2}\right)^2}-\left(z-\tfrac{L}{2}\right)\sqrt{R^2+\left(z-\tfrac{L}{2}\right)^2}+R^2\ln\left[\frac{z+\frac{L}{2}+\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}}{z-\frac{L}{2}+\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}}\right]-2zL\right\}.}$$

(*Note:* $-\left(z+\frac{L}{2}\right)^2+\left(z-\frac{L}{2}\right)^2 = -z^2-zL-\frac{L^2}{4}+z^2-zL+\frac{L^2}{4} = -2zL.$)

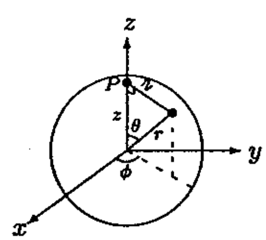
$$\mathbf{E} = -\nabla V = -\hat{\mathbf{z}}\frac{\partial V}{\partial z} = -\frac{\hat{\mathbf{z}}\rho}{4\epsilon_0}\left\{\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}+\frac{\left(z+\frac{L}{2}\right)^2}{\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}}-\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}-\frac{\left(z-\frac{L}{2}\right)^2}{\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}}\right.$$

$$\left.+R^2\underbrace{\left[\frac{1+\frac{z+\frac{L}{2}}{\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}}}{z+\frac{L}{2}+\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}}-\frac{1+\frac{z-\frac{L}{2}}{\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}}}{z-\frac{L}{2}+\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}}\right]}_{\frac{1}{\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}}-\frac{1}{\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}}}-2L\right\}$$

$$\mathbf{E} = -\frac{\hat{\mathbf{z}}\rho}{4\epsilon_0}\left\{2\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}-2\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}-2L\right\}$$

$$= \boxed{\frac{\rho}{2\epsilon_0}\left[L-\sqrt{R^2+\left(z+\frac{L}{2}\right)^2}+\sqrt{R^2+\left(z-\frac{L}{2}\right)^2}\right]\hat{\mathbf{z}}.}$$

Problem 2.28



Orient axes so P is on z axis.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{\rho}{\mathscr{r}}d\tau. \qquad \left\{\begin{array}{l}\text{Here } \rho \text{ is constant, } d\tau = r^2\sin\theta\, dr\, d\theta\, d\phi,\\ \mathscr{r} = \sqrt{z^2+r^2-2rz\cos\theta}.\end{array}\right.$$

$$V = \frac{\rho}{4\pi\epsilon_0}\int\frac{r^2\sin\theta\, dr\, d\theta\, d\phi}{\sqrt{z^2+r^2-2rz\cos\theta}}\ ;\ \int_0^{2\pi}d\phi = 2\pi.$$

$$\int_0^{\pi}\frac{\sin\theta}{\sqrt{z^2+r^2-2rz\cos\theta}}d\theta = \frac{1}{rz}\left(\sqrt{r^2+z^2-2rz\cos\theta}\right)\Big|_0^{\pi} = \frac{1}{rz}\left(\sqrt{r^2+z^2+2rz}-\sqrt{r^2+z^2-2rz}\right)$$

$$= \frac{1}{rz}\left(r+z-|r-z|\right) = \left\{\begin{array}{l}2/z\ ,\ \text{if } r<z,\\ 2/r\ ,\ \text{if } r>z.\end{array}\right\}$$

$$\therefore V = \frac{\rho}{4\pi\epsilon_0}\cdot 2\pi\cdot 2\left\{\int_0^z \frac{1}{z}r^2dr + \int_z^R \frac{1}{r}r^2dr\right\} = \frac{\rho}{\epsilon_0}\left\{\frac{1}{z}\frac{z^3}{3} + \frac{R^2-z^2}{2}\right\} = \frac{\rho}{2\epsilon_0}\left(R^2 - \frac{z^2}{3}\right).$$

But $\rho = \frac{q}{\frac{4}{3}\pi R^3}$, so $V(z) = \frac{1}{2\epsilon_0}\frac{3q}{4\pi R^3}\left(R^2 - \frac{z^2}{3}\right) = \frac{q}{8\pi\epsilon_0 R}\left(3 - \frac{z^2}{R^2}\right)$; $\boxed{V(r) = \frac{q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right).}$ ✓

Problem 2.29

$\nabla^2 V = \frac{1}{4\pi\epsilon_0}\nabla^2\int\left(\frac{\rho}{\mathscr{r}}\right)d\tau = \frac{1}{4\pi\epsilon_0}\int\rho(\mathbf{r}')\left(\nabla^2\frac{1}{\mathscr{r}}\right)d\tau$ (since ρ is a function of $\mathbf{r}'$, not $\mathbf{r}$)

$= \frac{1}{4\pi\epsilon_0}\int\rho(\mathbf{r}')[-4\pi\delta^3(\mathbf{r}-\mathbf{r}')]\,d\tau = -\frac{1}{\epsilon_0}\rho(\mathbf{r})$. ✓

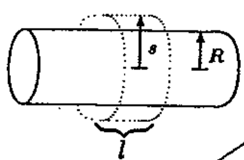
Problem 2.30.

(a) Ex. 2.4: $\mathbf{E}_{\text{above}} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}}$; $\mathbf{E}_{\text{below}} = -\frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}}$ ($\hat{\mathbf{n}}$ always pointing *up*); $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$. ✓

Ex. 2.5: At each surface, $E = 0$ one side and $E = \frac{\sigma}{\epsilon_0}$ other side, so $\Delta E = \frac{\sigma}{\epsilon_0}$. ✓

Prob. 2.11: $\mathbf{E}_{\text{out}} = \frac{\sigma R^2}{\epsilon_0 r^2}\hat{\mathbf{r}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{r}}$; $\mathbf{E}_{\text{in}} = 0$; so $\Delta\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{r}}$. ✓

(b)



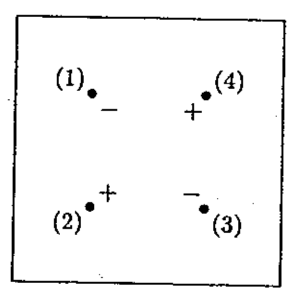
Outside: $\oint \mathbf{E}\cdot d\mathbf{a} = E(2\pi s)l = \frac{1}{\epsilon_0}Q_{\text{enc}} = \frac{1}{\epsilon_0}\sigma(2\pi R)l \Rightarrow \mathbf{E} = \frac{\sigma}{\epsilon_0}\frac{R}{s}\hat{\mathbf{s}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{s}}$ (at surface).

Inside: $Q_{\text{enc}} = 0$, so $\mathbf{E} = 0$. $\therefore \Delta\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{s}}$. ✓

(c) $V_{\text{out}} = \frac{R^2\sigma}{\epsilon_0 r} = \frac{R\sigma}{\epsilon_0}$ (at surface); $V_{\text{in}} = \frac{R\sigma}{\epsilon_0}$; so $V_{\text{out}} = V_{\text{in}}$. ✓

$\frac{\partial V_{\text{out}}}{\partial r} = -\frac{R^2\sigma}{\epsilon_0 r^2} = -\frac{\sigma}{\epsilon_0}$ (at surface); $\frac{\partial V_{\text{in}}}{\partial r} = 0$; so $\frac{\partial V_{\text{out}}}{\partial r} - \frac{\partial V_{\text{in}}}{\partial r} = -\frac{\sigma}{\epsilon_0}$. ✓

Problem 2.31



(a) $V = \frac{1}{4\pi\epsilon_0}\sum\frac{q_i}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left\{\frac{-q}{a} + \frac{q}{\sqrt{2}a} + \frac{-q}{a}\right\} = \frac{q}{4\pi\epsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$.

$\therefore W_4 = qV = \boxed{\frac{q^2}{4\pi\epsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right).}$

(b) $W_1 = 0$, $W_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{-q^2}{a}\right)$; $W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right)$; $W_4 =$ (see (a)).

$W_{\text{tot}} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a}\left\{-1 + \frac{1}{\sqrt{2}} - 1 - 2 + \frac{1}{\sqrt{2}}\right\} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{2q^2}{a}\left(-2 + \frac{1}{\sqrt{2}}\right).}$

Problem 2.34

(a) $W = \frac{\epsilon_0}{2}\int E^2\,d\tau$. $\quad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\ (a < r < b)$, zero elsewhere.

$$W = \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2 \int_a^b \left(\frac{1}{r^2}\right)^2 4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0}\int_a^b \frac{1}{r^2} = \boxed{\frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right).}$$

(b) $W_1 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{a}$, $W_2 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{b}$, $\quad \mathbf{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}\ (r > a)$, $\mathbf{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{-q}{r^2}\hat{\mathbf{r}}\ (r > b)$. So $\mathbf{E}_1\cdot\mathbf{E}_2 = \left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{-q^2}{r^4}$, $(r > b)$, and hence $\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 q^2\int_b^\infty \frac{1}{r^4}4\pi r^2 dr = -\frac{q^2}{4\pi\epsilon_0 b}$.

$W_{\text{tot}} = W_1 + W_2 + \epsilon_0\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau = \frac{1}{8\pi\epsilon_0}q^2\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{b}\right) = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right).\checkmark$

Problem 2.35

(a) $\boxed{\sigma_R = \frac{q}{4\pi R^2};\ \sigma_a = \frac{-q}{4\pi a^2};\ \sigma_b = \frac{q}{4\pi b^2}.}$

(b) $V(0) = -\int_\infty^0 \mathbf{E}\cdot d\mathbf{l} = -\int_\infty^b\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_b^a (0)dr - \int_a^R\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_R^0 (0)dr = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{q}{b}+\frac{q}{R}-\frac{q}{a}\right).}$

(c) $\boxed{\sigma_b \to 0}$ (the charge "drains off"); $V(0) = -\int_\infty^a (0)dr - \int_a^R\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_R^0 (0)dr = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}-\frac{q}{a}\right).}$

Problem 2.36

(a) $\boxed{\sigma_a = -\frac{q_a}{4\pi a^2};}\ \boxed{\sigma_b = -\frac{q_b}{4\pi b^2};}\ \boxed{\sigma_R = \frac{q_a+q_b}{4\pi R^2}.}$

(b) $\boxed{\mathbf{E}_{\text{out}} = \frac{1}{4\pi\epsilon_0}\frac{q_a+q_b}{r^2}\hat{\mathbf{r}},}$ where $\mathbf{r}$ = vector from center of large sphere.

(c) $\boxed{\mathbf{E}_a = \frac{1}{4\pi\epsilon_0}\frac{q_a}{r_a^2}\hat{\mathbf{r}}_a,\quad \mathbf{E}_b = \frac{1}{4\pi\epsilon_0}\frac{q_b}{r_b^2}\hat{\mathbf{r}}_b,}$ where $\mathbf{r}_a$ ($\mathbf{r}_b$) is the vector from center of cavity a (b).

(d) $\boxed{\text{Zero.}}$

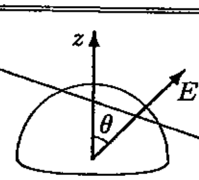
(e) σ_R changes (but not σ_a or σ_b); $\mathbf{E}_{\text{outside}}$ changes (but not $\mathbf{E}_a$ or $\mathbf{E}_b$); force on q_a and q_b still zero.

Problem 2.37

Between the plates, $E = 0$; outside the plates $E = \sigma/\epsilon_0 = Q/\epsilon_0 A$. So

$$P = \frac{\epsilon_0}{2}E^2 = \frac{\epsilon_0}{2}\frac{Q^2}{\epsilon_0^2 A^2} = \boxed{\frac{Q^2}{2\epsilon_0 A^2}.}$$

Problem 2.38



Inside, $\mathbf{E} = 0$; outside, $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$; so

$\mathbf{E}_{\text{ave}} = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{\mathbf{r}}$; $\ f_z = \sigma(E_{\text{ave}})_z$; $\ \sigma = \frac{Q}{4\pi R^2}$.

$F_z = \int f_z\,da = \int\left(\frac{Q}{4\pi R^2}\right)\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\right)\cos\theta\,R^2\sin\theta\,d\theta\,d\phi$

$= \frac{1}{2\epsilon_0}\left(\frac{Q}{4\pi R}\right)^2 2\pi\int_0^{\pi/2}\sin\theta\cos\theta\,d\theta = \frac{1}{\pi\epsilon_0}\left(\frac{Q}{4R}\right)^2\left(\frac{1}{2}\sin^2\theta\right)\Big|_0^{\pi/2} = \frac{1}{2\pi\epsilon_0}\left(\frac{Q}{4R}\right)^2 = \boxed{\frac{Q^2}{32\pi R^2\epsilon_0}.}$