2) SOLUTION 2

$$d\vec{r} = \hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz.$$

by definition, $\displaystyle df = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial \phi}d\phi + \frac{\partial f}{\partial z}dz$

and $\displaystyle df = \vec{\nabla} f \cdot d\vec{r} = (\vec{\nabla} f)_r\,dr + (\vec{\nabla} f)_\phi \overset{r}{\downarrow} d\phi + (\vec{\nabla} f)_z\,dz$

So, $\displaystyle \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial \phi}d\phi + \frac{\partial f}{\partial z}dz = (\vec{\nabla} f)_r\,dr + (\vec{\nabla} f)_\phi \overset{r}{\downarrow} d\phi + (\vec{\nabla} f)_z\,dz$

Thus, $\displaystyle (\vec{\nabla} f)_r = \frac{\partial f}{\partial r}$, $\displaystyle (\vec{\nabla} f)_\phi = \frac{1}{r}\frac{\partial f}{\partial \phi}$, $\displaystyle (\vec{\nabla} f)_z = \frac{\partial f}{\partial z}$

i.e., $\displaystyle \vec{\nabla} f(r,\phi,z) = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$