

# Physics Comprehensive Exam - Spring 2006

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## Electricity & Magnetism

March 22, 2006  
9:00 am – 2:00 pm

Instructions:

1. Work **four** of the five problems in Electricity & Magnetism. **DO NOT WORK ALL FIVE!**
2. On this cover sheet, circle the problems that you have chosen to hand in.
3. Clearly write your code symbol - NOT YOUR NAME - at the top of each sheet of paper you hand in.
4. Start each new problem at the top of a fresh sheet of paper.
5. Explain your reasoning whenever possible.

GOOD LUCK!

Name:

Code Symbol:

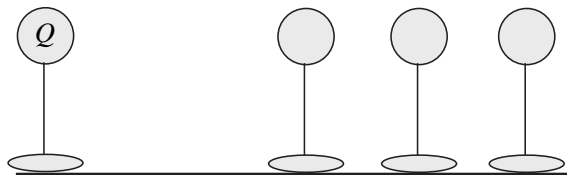
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Electricity & Magnetism problems:    1    2    3    4    5

# Electricity & Magnetism 1

## Charge Sharing.

Four identical conducting balls are attached to insulating supports that sit on the floor as shown below. One ball has charge  $Q$ ; its support is fixed in space. The other three balls are uncharged but their supports can be moved around. Describe a procedure (that involves only moving and/or bringing balls into contact) that will leave the  $+Q$  ball with its full charge and give the three originally uncharged balls charges  $q$ ,  $-q/2$ , and  $-q/2$ . You may assume that  $q < Q$ .



## *Electricity & Magnetism 2*

### **No Magnetic Force.**

Maxwell showed that the magnetic force exerted on everything that lies inside a closed surface  $S$  is

$$\mathbf{F} = \frac{1}{\mu_0} \int_S dS [(\hat{\mathbf{n}} \cdot \mathbf{B})\mathbf{B} - \frac{1}{2}\hat{\mathbf{n}}(\mathbf{B} \cdot \mathbf{B})].$$

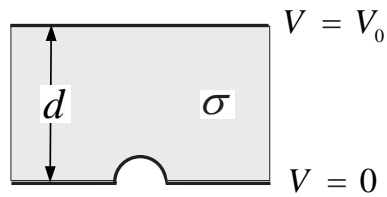
Use this formula to prove that no isolated, time-independent current distribution  $\mathbf{j}(\mathbf{r})$  can exert a force on itself.

## Electricity & Magnetism 3

### Current Flow to a Bump.

A voltage difference  $V_0$  causes a steady current to flow from the top conductor to the bottom conductor (in the sketch below) through an Ohmic medium with conductivity  $\sigma$ . Find an approximate expression for the current  $I$  that flows into the hemispherical bump (radius  $R$ ) portion of the lower conductor. Assume that  $d \gg R$ .

Hint: A grounded spherical conductor in an external field  $\mathbf{E}_0$  acquires a surface charge density  $\Sigma = 3\epsilon_0 E_0 \cos \theta$  where  $\theta$  is the polar angle measured from  $\hat{\mathbf{E}}$ .



## *Electricity & Magnetism 4*

### **Torque on a Cylinder.**

An infinite cylinder (radius  $a$ ) coaxial with the  $z$ -axis is uniformly filled with charge with density  $\rho$ . Every bit of charge feels a uniform magnetic field  $\mathbf{B} = B\hat{z}$ . Find the instantaneous torque per length that acts on the cylinder when the magnitude of the magnetic field is changed according to some function  $B(t)$ .

## *Electricity & Magnetism 5*

### **Rotating Triangle Radiation.**

Three identical point charges  $q$  are glued to the corners of an equilateral triangle that lies in the  $x$ - $y$  plane. The charges rotate with constant angular velocity  $\omega$  around the  $z$ -axis, which passes through the center of the triangle.

- (a) Does this system produce electric dipole radiation?
- (b) Does this system produce magnetic dipole radiation?
- (c) Does this system produce electric quadrupole radiation?

# Physics Comprehensive Exam - Spring 2006

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## Quantum Mechanics

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9:00 am – 2:00 pm

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Name:

Code Symbol:

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Quantum Mechanics problems:    1    2    3    4    5

# *Quantum Mechanics 1*

## **One Dimensional Well.**

A particle of mass  $m$  moves in one dimension in a square well with walls of infinite height a distance  $L$  apart. The particle is known to be in a state corresponding to the equal probability superposition of the two lowest energy eigenstates of the system. Find the probability as a function of time that the particle will be found in the right-hand half of the well.

## Quantum Mechanics 2

### Energy Shift due to Finite Size of Nucleus.

Consider the effect on the ground state of hydrogen atom caused by the finite size of the proton. Assume that the proton is a uniformly charged sphere of radius  $R = 10^{-13}$ cm. The ground state wave function of the hydrogen atom is

$$|1, 0, 0\rangle = \sqrt{\frac{1}{\pi a_0^3}} e^{-r/a_0}$$

and the Bohr radius is  $a_0 = 0.53 \times 10^{-10}$ m.

- (a) Calculate the potential  $U(r)$  produced by the finite sized proton.
- (b) Calculate the first order shift to the ground state energy of the hydrogen atom.

## Quantum Mechanics 3

### Spin 1/2 Particles.

Two non-identical spin 1/2 particles with magnetic moments  $\mu_1 = -g\mathbf{S}_1$  and  $\mu_2 = -g\mathbf{S}_2$ , respectively, are placed in a region of free external fields. Their interaction energy is given by  $H = \gamma\mathbf{S}_1 \cdot \mathbf{S}_2$ .

- (a) Find the eigenvalues and eigenstates of this Hamiltonian.
- (b) A very weak external magnetic field  $\mathbf{B} = B_0\hat{\mathbf{z}}$  is introduced. The interaction between the particles and the field is much weaker than their mutual interaction. Find the first order correction to the eigenvalues.

## Quantum Mechanics 4

### Quantum Symmetric Top.

Consider a symmetric top (with the moment of inertia  $I$ ) spinning about its symmetry axis with non-zero angular momentum  $\mathbf{J}$ . Assume that the top is in the presence of an external magnetic field  $\mathbf{B}$ , so that its Hamiltonian is

$$H = \frac{\mathbf{J}^2}{2I} - \boldsymbol{\mu} \cdot \mathbf{B}.$$

The magnetic moment of the top is  $\boldsymbol{\mu} = (G\mu_B/\hbar)\mathbf{J}$ , where  $G > 0$  and  $\mu_B$  is the Bohr magneton. Assume that the top has an integer total angular momentum quantum number.

- (a) What are the ground state energy and the energy of the first excited state of the spinning top, when  $B = B_0\hat{\mathbf{z}}$ ,  $B_0 > 0$ ?
- (b) Assume that  $\mathbf{B}$  becomes time dependent with  $B(t) = B_0\hat{\mathbf{z}}$  for  $t < 0$  and  $B(t) = B_0\hat{\mathbf{z}} + \Delta B\hat{\mathbf{x}}e^{-\lambda t}$  for  $t > 0$ , where  $\Delta B \ll B_0$  and  $\lambda > 0$ . If the top is in its first excited state for  $t < 0$ , find the probability that the top is in the ground state for  $t > 0$ .

## Quantum Mechanics 5

### Identical Fermions.

The Hamiltonian for two interacting identical fermions (spin 1/2) in one dimension is

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{m\omega^2}{2}(x_2 - x_1)^2.$$

- (a) What is the energy spectrum?
- (b) What are the corresponding eigenfunctions (including the spin state)?