Measuring Spatial Chirp and Pulse-Front Tilt in Ultrashort Pulses Using Single-Shot FROG

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Spatial-chirp (in which the average wavelength of the pulse varies spatially across the beam) and pulse-front tilt (in which the pulse intensity fronts are not perpendicular to the propagation vector) are common spatio-temporal distortions in ultrashort pulses. Devices such as pulse compressors deliberately introduce considerable spatial chirp and pulse-front tilt into pulses only to, in principle, remove them afterward. However, even small misalignments leave residual distortions into the pulse. Unfortunately, convenient diagnostics are not available for these distortions. We show that spatial chirp is easily measured by any single-shot second-harmonic-generation frequency-resolved-optical-gating (SHG FROG) device, including its extremely simple variation, GRENOUILLE. Specifically, the ordinarily symmetrical (untilted) SHG FROG trace develops a *tilt* in the presence of spatial chirp and which is twice the spatial chirp. We also show that GRENOUILLE easily measures pulse-front tilt: it yields a displacement along the delay axis in GRENOUILLE measurements.

The inversion formulas are very simple. Because the single-shot SHG FROG trace and spatio-spectral diagnostics for spatial chirp both involve plots of intensity vs. frequency and position, the FROG-trace tilt is naturally related to the spatial chirp. Indeed, the single-shot FROG or GRENOUILLE trace tilt is approximately *twice* the spatial chirp when plotted vs. frequency and *one half* when plotted vs. wavelength (Fig.1). Pulse-front tilt measurement involves simply measuring the GRENOUILLE trace displacement (Fig.1). These trace distortions can then be removed and the pulse retrieved using the usual algorithm, and the spatio-temporal distortions can be included in the resulting pulse intensity and phase.

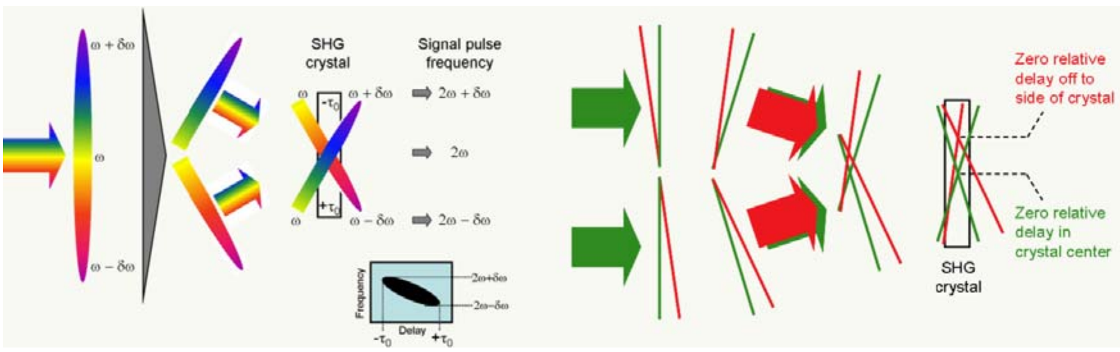

Fig. 1. Spatial chirp tilts the trace (left), and pulse-front tilt translates the trace (right) in GRENOUILLE measurements.

To see the effect of spatial chirp on single-shot FROG measurements (see Fig. 3), we begin with the usual expression for an SHG FROG trace, including the carrier frequencies of the two pulses:

$$I_{FROG}^{SHG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E(t)\, E(t-\tau)\, \exp[-i(\omega - 2\omega_0)t]\, dt \right|^2 \qquad (1)$$

In single-shot FROG techniques, two replicas of the pulse are crossed at a large angle, and delay is mapped onto position, $\tau = \alpha x$, where $\alpha = 2\sin(\theta/2)/c$. This yields: $I_{FROG}^{SHG}(\omega,\alpha x)$.

The frequency of a spatially chirped pulse can be expressed as $\omega(x) = \omega_0 + \xi x$, yielding:

$$I_{FROG}^{SHG}(\omega,\alpha x) = \left| \int_{-\infty}^{\infty} E(t)\, E(t-\tau)\, \exp[-i(\omega - 2\omega_0 - 2\xi x)t]\, dt \right|^2 \qquad (2)$$

This is simply: $I_{FROG}^{SHG}(\omega - 2\xi x, \alpha x)$. Thus the SHG FROG trace, which is normally symmetrical with respect to delay, develops tilt in the presence of spatial chirp. No other effect causes such asymmetry, so this is a simple and clear measure of spatial chirp.

To vary the spatial chirp of a pulse, we placed mirrors between the last two prisms of a pulse compressor, deflecting the pulse to two additional mirrors mounted on a translation stage. By translating the latter two mirrors, we were able to align and misalign the compressor, obtaining positive, zero, or negative spatial chirp.

We have also made independent measurements of spatial chirp by measuring spatio-spectral plots, obtained by sending the beam through a carefully aligned imaging spectrometer (ordinary spectrometers are not usually good diagnostics for spatial chirp due to aberrations in them that mimic the effect) and spatially resolving the output on a 2D camera, which yields a tilted image (spectrum vs. position) in the presence of spatial-chirp. We find very good agreement between this measurement of spatial chirp and that from GRENOUILLE measurements (Fig.2).

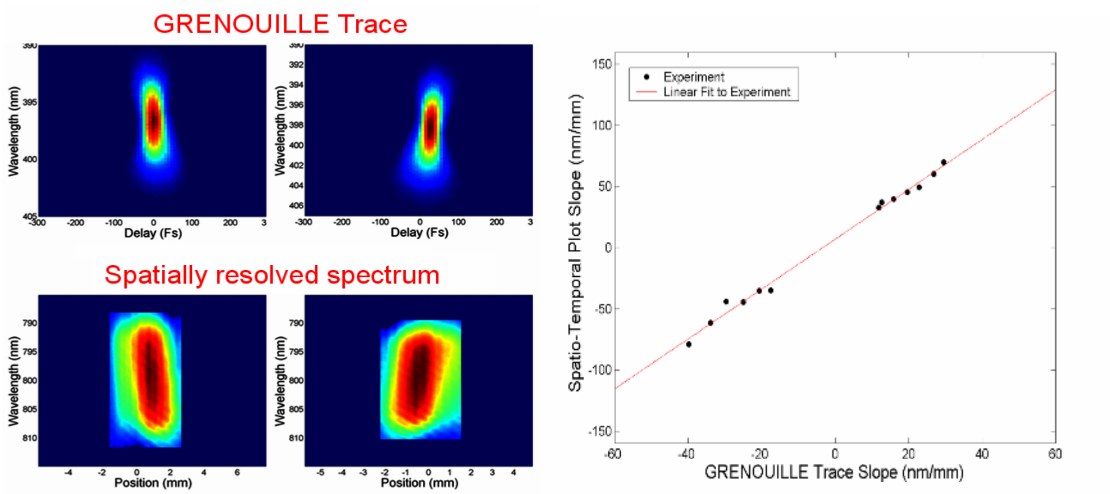

Fig. 2. Experimental GRENOUILLE traces (left top) and corresponding spatio-spectral plots (left bottom) for pulses with positive and negative spatial chirp. The tilt in GRENOUILLE traces reveals the magnitude and sign of spatial chirp. Right: Slopes of GRENOUILLE traces and corresponding spectrum vs. position slopes for various amounts of spatial chirp.

In the presence of pulse-front tilt, the expression for the GRENOUILLE trace becomes:

$$I_{SHGFROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E(t+\tau_0+\zeta x)\exp[i\omega_0 t]\, E(t-\tau_0+\zeta x-\tau)\exp[i\omega_0(t-\tau)]\exp[-i\omega t]\, dt \right|^2 \quad (3)$$

which can be simplified to $I_{SHGFROG}(\omega,\tau+2\tau_0)$.

The effect of pulse-front tilt is thus to add a constant delay $\tau_0 = \zeta x_0$ to the GRENOUILLE trace. Thus, the trace will be off-center by an amount proportional to the pulse-front tilt.

To vary the pulse-front tilt of a pulse, we placed the last prism of a pulse compressor on a rotary stage. By rotating the stage we were able to align and misalign the compressor, obtaining positive, zero, or negative pulse-front tilt. Figure 3 shows theoretical and experimental values of pulse-front tilt in our experiments (right) and some experimental GRENOUILLE traces for different amounts of pulse-front tilt (left). We find very good agreement between theoretical values of pulse-front tilt and that from GRENOUILLE measurements.

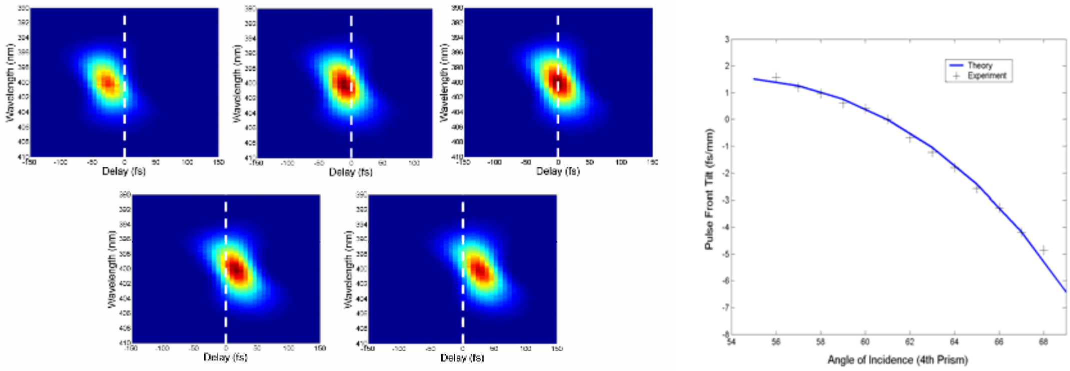

Fig.3. Measured GRENOUILLE traces for pulses with very negative, slightly negative, zero, and slightly positive, and very positive pulse-front tilt. Notice that the trace displacement is proportional to the pulse-front tilt (left). Theoretically predicted pulse-front tilt and the experimentally measured pulse-front tilt using GRENOUILLE (right).

In short, SHG FROG and GRENOUILLE provide additional important pulse information, beyond the pulse intensity and phase vs. time and frequency. They also sensitively measure otherwise unavailable spatio-temporal distortions, spatial chirp and pulse-front tilt. Indeed, we have found that GRENOUILLE is the most sensitive measure of pulse-front tilt available.