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Error Bars in Intensity and Phase Measurements of Ultrashort Laser Pulses

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How does one place error bars on a measurement of an ultrashort laser pulse?

Because an autocorrelation trace typically corresponds to many—often quite different—intensities, even a perfect noise-free autocorrelation results in a large—and unknown—uncertainty in the shape of the pulse's intensity vs. time. Autocorrelation thus has a type of "internal noise" that occurs even in the absence of measurement error, so it makes no sense to attempt to place error bars on such a reconstruction of the pulse.

Of course, FROG retrieves the full pulse intensity and phase without assumptions. But just how accurate is a given FROG measurement of a pulse? Unfortunately, direct computation of error bars in FROG (or any experiment) involves a tedious—and often questionable—accounting of all the known sources or error.

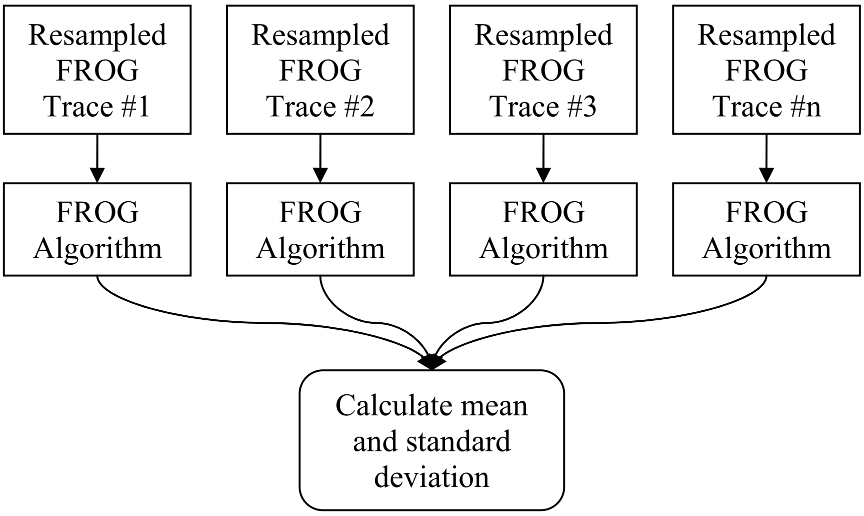

Figure 1: A Schematic of the bootstrap process. Each of the resampled traces is run through the FROG algorithm and the mean and standard deviation of each intensity and phase point in the retrievals is calculated, yielding the intensity and phase point and error bar, respectively.

We solve this problem using the "bootstrap" method, a well-established statistical method. The bootstrap method involves taking the data set of M points, choosing a new set of M points from it, one at a time—but *with replacement* (resulting in some points occurring more than once and others not at all)—and then running the relevant parameter-determination algorithm on this new set of points. If this process is repeated numerous times on a given data set, each desired parameter will be determined numerous times. It has been shown that, when the data over-determine the parameters (i.e., when there are many more data points than parameters), the set of parameter values obtained by this procedure is generally the correct probability distribution for the parameters. In particular, the mean and standard deviation of each parameter's distribution are the measured parameter and the error bar, respectively. This method is called the bootstrap method because it appears that one is getting something for nothing ("pulling oneself up by his own bootstraps"), but this is not the case, and, in fact, the method can be quite computation intensive. With the use of modern computers, however, the required computations are not prohibitive, and the computations we perform require at most a few minutes on a personal computer.

Applying this approach to ultrashort-pulse measurement simply involves running the FROG retrieval algorithm on the order of 10 to 100 times on the measured FROG trace, but each time with only a subset of points, chosen at random as described above, and tabulating the statistics of the retrieved intensity and phase values obtained during these runs (See Fig. 1). The mean intensity and phase values for each time and frequency are then the measured values, and the standard deviations yield the error bars. This works because the FROG trace over-determines the pulse, that is, contains many more points than the resulting intensity and phase. Figure 2 shows a retrieved intensity and phase with error bars computed using the bootstrap procedure for a trace with noise added numerically. We have shown that the error bars computed this way are quite reasonable.

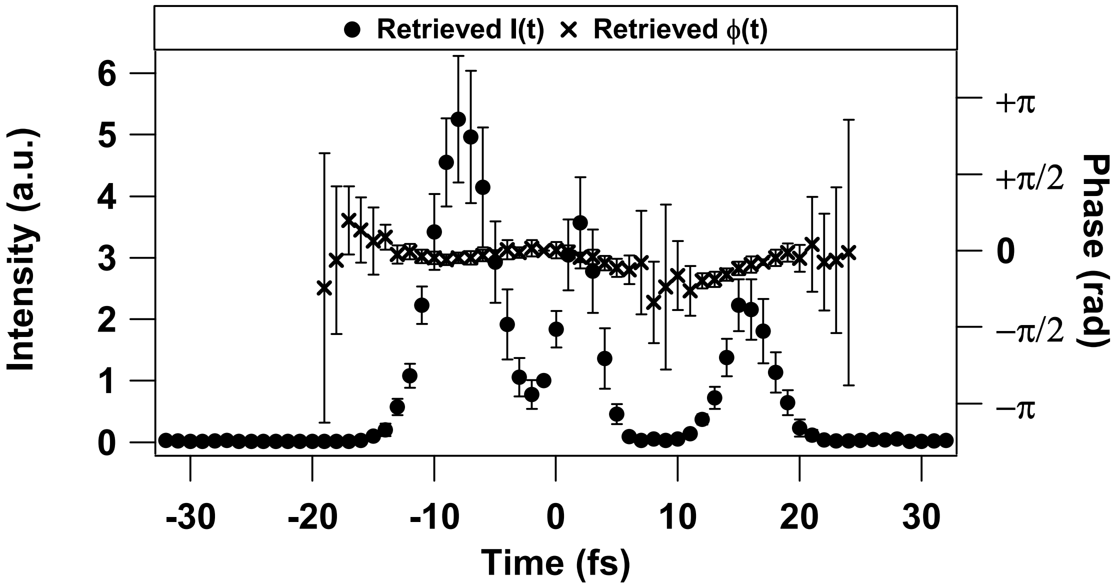

Figure 2. Retrieved intensity and phase and error bars of a theoretical pulse with 1% additive noise introduced numerically to the FROG trace. We have also used the computed error bars to solve another problem: to "phase-blank," which here means the removal of any phase points when the intensity approaches zero and whose error thus exceeds 2π.