Heat current fluctuations in quantum wires

I. V. Krive,[1,2] E. N. Bogachek,[2] A. G. Scherbakov,[2] and Uzi Landman[2]
[1]*B. I. Verkin Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine*
[2]*School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430*
(Received 18 December 2000; revised manuscript received 25 June 2001; published 8 November 2001)

Noise in the thermal current through a ballistic quantum wire is considered both for noninteracting and interacting particles. It is shown that for a perfect quantum wire the equilibrium thermal (Johnson-Nyquist) noise does not depend on the statistics of the heat carriers. In contrast, the nonequilibrium noise, produced by the temperature difference between the heat baths, is different for fermions and bosons. The general expressions which are obtained, are used in calculations of the noise power of the thermal current through a Luttinger liquid wire connected to reservoirs of noninteracting electrons.

DOI: 10.1103/PhysRevB.64.233304 PACS number(s): 72.10.Bg, 73.23.−b, 85.35.Be

I. INTRODUCTION

While electric current fluctuations in mesoscopic systems are theoretically well studied (see reviews in Refs. 1 and 2), less attention has been paid until quite recently to fluctuation phenomena in the thermal transport through conducting and dielectric quantum wires. A theory of electronic and phononic heat transport through point contacts has been developed in the mid-1980s using the kinetic equation approach.[3,4] Thermoelectric effects in ballistic wires can be described[5] within the framework of the Landauer-Buttiker formalism,[6] and employing this approach electronic heat transport in two- and three-dimensional quantum wires has been considered.[7–9] Recently the problem of ballistic heat transport was reexamined and applied to phonon energy transport in suspended dielectric wires,[10–12] and to plasmon transport in a Luttinger liquid.[13,14] Experimental attempts to measure heat transport in dielectric wires in the quantum regime (see Ref. 10 and references therein) resulted in observation[15] of the quantized value of the thermal conductance $K_0(T)=(\pi^2/3)(k_B^2T/h)$ (T is the temperature and k_B is the Boltzmann constant) in a suspended dielectric nanostructure.

Most theoretical studies of heat transport in quantum wires dealt mainly with the mean-energy current. Fluctuations in the phononic current were considered in Ref. 12, where a general expression for the variance of the energy current has been derived. In this paper we investigate the noise in the heat current for Fermionic and Bosonic systems in the Landauer-Buttiker approach. We show that the equilibrium thermal (Johnson-Nyquist) noise in a perfect quantum wire does not depend on the statistics of the heat carriers. In contrast, we find that the nonequilibrium noise produced by the temperature difference between the heat reservoirs (hot and cold leads) is different for bosons and fermions, and that the contribution of the ''Fermionic'' nonequilibrium noise to the thermal noise is positive while the ''Bosonic'' contribution is negative (for a similar behavior pertaining to the noise power of the electric current see Refs. 2 and 16). At all temperatures the nonequilibrium noise $P_{\Delta T}$ is much smaller than the Johnson-Nyquist noise $P_{\rm JN}$–even in the most favorable situation, $|P_{\Delta T}|/P_{\rm JN}\leqslant 0.3$ for bosons and $P_{\Delta T}/P_{\rm JN}\leqslant 0.1$ for fermions.

As an example, we consider thermal current fluctuations in a Luttinger liquid (LL) wire adiabatically connected to the electron reservoirs. In the absence of electron backscattering heat is transported in a LL by neutral noninteracting Bosonic excitations (plasmons). By modeling the leads as reservoirs of one-dimensional (1D) noninteracting electrons we calculate the noise power of the heat current through a LL constriction, and show that it is dominated by the Johnson-Nyquist noise generated by the leads.

II. THERMAL AND NONEQUILIBRIUM NOISE IN QUANTUM WIRES

Current fluctuations are characterized by the noise power[1,2]

$$P(\omega)=2\int_{-\infty}^{\infty}dt\cdot e^{i\omega t}\langle\Delta I(t)\Delta I(0)\rangle, \tag{1}$$

where $\Delta I(t)=I(t)-\overline{I}$; here $\overline{I}$ is the average current and the angular brackets indicate an ensemble average. In our problem $I(t)$ denotes the energy current operator. With the use of scattering theory,[1,2] it is straightforward to represent the noise power of the thermal current fluctuations at $\omega\to 0$ in the form [for an analogous formula for the electric current noise see Eq. (61) in Ref. 2]

$$\begin{aligned}P_Q^{(f,b)}=&\frac{1}{\pi\hbar}\int_0^{\infty}dE(E-\mu_{f,b})^2\{T_t(E)[f_L^{(f,b)}(1\mp f_L^{(f,b)})+f_R^{(f,b)}\\&\times(1\mp f_R^{(f,b)})]\pm T_t(E)[1-T_t(E)]\\&\times(f_L^{(f,b)}-f_R^{(f,b)})^2\}.\end{aligned} \tag{2}$$

Here $T_t(E)$ is the transmission coefficient, $f_{L,R}^{(f,b)}$ is the distribution function of fermions (f) or bosons (b) in the left (L) and right (R) heat baths (leads). The upper and lower signs in Eq. (2) correspond to Fermi and Bose statistics, respectively. In deriving Eq. (2) we neglect for $k_BT\ll\mu_f$ the temperature dependence of the chemical potentials in the left and right leads, $\mu_{L,R}$, and take them to be equal, i.e., $\mu_L\simeq\mu_R\equiv\mu_f$. For neutral bosons (e.g., phonons, plasmons) $\mu_b=0$.

The first two terms inside the square brackets in Eq. (2) represent the equilibrium (Johnson-Nyquist) noise.[2,17] They can be rewritten in terms of the thermal conductance, i.e.,

$$P_{\rm JN}=2k_BT_L^2K^{(f,b)}(T_L)+2k_BT_R^2K^{(f,b)}(T_R), \tag{3}$$

where the thermal conductance $K(T)$ is given by[5]

$$K^{(f,b)}(T)=\frac{1}{hT}\int_0^\infty dE(E-\mu_{f,b})^2\left(-\frac{\partial f^{(f,b)}}{\partial E}\right)T_t(E). \tag{4}$$

It is of interest to note that for an energy-independent transmission probability T_t the thermal conductance and the Johnson-Nyquist noise [Eq. (3)], do not depend on the statistics of the carriers,[18,19] and they can be expressed in terms of the universal quantum of thermal conductance, $K_0(T)$.

The last term in Eq. (2) describes the nonequilibrium noise $P_{\Delta T}$. It vanishes when the temperatures of the left and right heat baths are the same, $T_L=T_R$. For electric current fluctuations the analogous term in the noise power is called shot noise. It can be separated by setting $T_{L,R}\to 0$ and applying nonzero bias voltage, $V\neq 0$. In our case the nonequilibrium transport is produced by finite temperature difference, $\Delta T=T_R-T_L$. It is evident that the nonequilibrium noise $P_{\Delta T}$ is small for a small temperature difference, $P_{\Delta T}\sim(\Delta T)^2$.

To estimate the contribution of $P_{\Delta T}$ to the total noise power we assume that the transmission coefficient does not depend on energy. One can expect a maximal effect of the nonequilibrium state when $T_{L(R)}\gg T_{R(L)}$, and in this case

$$P_Q^{(f,b)}\simeq P_{\rm JN}(T)\{1\pm(1-T_t)A_{f,b}\}, \tag{5}$$

where T is the temperature of the hot lead, and $P_{\rm JN}(T)$ is the corresponding thermal noise given by Eq. (3). In Eq. (5) the upper sign corresponds to fermions and the lower sign to bosons. The factor $A_{f,b}$ determines the maximal contribution of the nonequilibrium noise to the total noise power

$$A_f=-1+\frac{9}{\pi^2}\zeta(3)\simeq 0.1, \tag{6}$$

$$A_b=1-\frac{6}{\pi^2}\zeta(3)\simeq 0.3, \tag{7}$$

where $\zeta(x)$ is the Riemann zeta function [$\zeta(3)\simeq 1.2$]. We observe that the nonequilibrium noise, in contrast to the Johnson-Nyquist noise, depends on statistics (see also Refs. 2 and 16). For Fermi statistics a large temperature difference slightly increases the noise power and it diminishes the noise for Bose particles. However, the effect is small—it is always smaller than 30% for bosons and 10% for fermions. In principle the nonequilibrium noise $P_{\Delta T}^{(f,b)}$ could be measured in situation when one can control the transmission probability T_t.

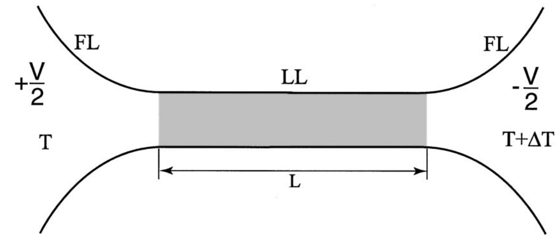

FIG. 1. Schematic of a Luttinger liquid (LL) nanowire of length L connected to Fermi liquid (FL) reservoirs kept at different temperatures and bias voltages.

III. THERMAL CURRENT FLUCTUATIONS IN A LUTTINGER LIQUID

We consider next thermal current fluctuations in a 1D interacting electron system described by the Luttinger liquid theory. It is known that for a perfect LL (without impurities) the electric current at $T\to 0$ is noiseless.[20] The same is true for a LL wire smoothly connected to the leads (see Fig. 1). Since, on the other hand, the heat current through a LL wire will always fluctuate because of the unavoidable thermal noise generated by the hot connected reservoirs (leads), we explore in the following whether such noise could be suppressed in a strong nonequilibrium situation.

In a LL without local scatterers heat is transported by neutral noninteracting Bosonic excitations (plasmons),[21] and we may use Eq. (2) to evaluate the noise power of the energy current. In contrast to electrons, plasmons are backscattered even by adiabatic leads,[13,14] and in the limit of strong repulsive interactions the backscattering of plasmons with wavelength $\lambda<L$ (L is the size of LL wire) is strong at the boundaries and the heat conductance is small. Consequently one may expect that nonequilibrium noise could diminish the thermal noise provided that $T_{L(R)}\gg T_{R(L)}$.

A quantitative estimate of the noise power can be obtained by modeling the leads as 1D noninteracting Fermi gases. In this case the transmission coefficient for the plasmons can be obtained by matching the wave functions of the plasmons (plane waves) at the boundaries between the LL wire and the electron reservoirs (Fermi liquid). Because of the mismatch of the velocities of plasmons in the leads (v_F) and in the LL wire ($s>v_F$), the plasmons are backscattered at the transition region between the Fermi liquid and the Luttinger liquid and the heat transport through the system is suppressed. The effect is analogous to the Kapitza resistance (see, e.g., Ref. 22). The transmission coefficient for the plasmons in our model reads[14]

$$T_t(\varepsilon)=\left[\cos^2\left(\frac{\varepsilon}{\Delta_L}\right)+\frac{1}{4}\left(g+\frac{1}{g}\right)^2\sin^2\left(\frac{\varepsilon}{\Delta_L}\right)\right]^{-1}. \tag{8}$$

Here $\Delta_L=\hbar s/L$ is the characteristic low-energy scale, s is the plasmon velocity, $g=v_F/s$ is the parameter which characterizes the strength of electron-electron interactions (the LL correlation parameter), v_F is the Fermi velocity, and $\varepsilon=E-E_F$. Results for the dependence of the equilibrium ($P_{\rm JN}$) and nonequilibrium ($P_{\Delta T}$) contributions to the total noise power of the thermal current (P_Q) on the temperature

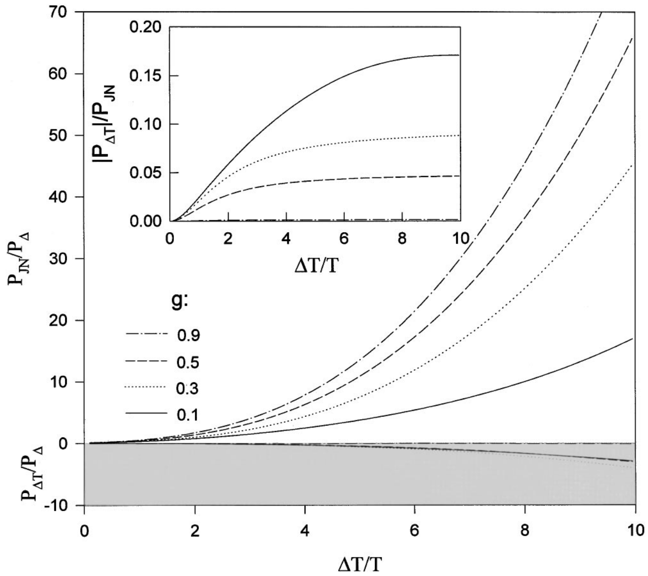

FIG. 2. The contribution of the equilibrium ($P_{\rm JN}$) and the nonequilibrium ($P_{\Delta T}$, in the gray part of the figure) noise to the total noise power of the thermal current, $P_Q = P_{\rm JN} + P_{\Delta T}$, in a LL constriction. $P_{\rm JN}$ and $P_{\Delta T}$ were normalized by a characteristic value $P_\Delta \equiv 2\pi\Delta_0^3/3\hbar$ ($\Delta_0 = \hbar v_F/L$). The plots in Fig. 2 correspond to $k_B T/\Delta_0 = 0.5$, with $T_L = T$, $T_R = T + \Delta T$. The inset shows the ratio $|P_{\Delta T}|/P_{\rm JN}$ for different values of the LL electron-electron interaction strength parameter g as a function of the fractional temperature difference $\Delta T/T$.

difference between the two leads, calculated from Eqs. (2) and (8), are displayed in Fig. 2 for different values of the LL correlation parameter g (we choose $k_B T/\Delta_0 = 0.5$ with $\Delta_0 = g\Delta_L$ and $T_L = T$). The relative contribution of the nonequilibrium noise (the ratio $|P_{\Delta T}|/P_{\rm JN}$ plotted in the inset) increases in wires with stronger electron-electron interactions (smaller values of g). We note that, as discussed above, the nonequilibrium noise ($P_{\Delta T}$) is much smaller than the equilibrium one ($P_{\rm JN}$). The specific form of the energy dependence of the transmission coefficient, Eq. (8), results in an additional suppression of $P_{\Delta T}$ (by a factor 2) in comparison with the maximum nonequilibrium noise given by Eqs. (5) and (7). Consequently, in a LL constriction the equilibrium thermal noise dominates the energy current fluctuations even in the situation ($T_R \equiv T + \Delta T \gg T \equiv T_L, g \ll 1$) when one could expect noticeable contributions from $P_{\Delta T}$ to the noise power.

IV. SUMMARY

In summary, we have shown that while the equilibrium thermal (Johnson-Nyquist) noise in perfect quantum wires does not depend on the statistics of the heat carriers, the nonequilibrium contribution to the total noise power is different for fermions than for bosons, with the former enhancing the noise power and the latter diminishing it; in both cases the nonequilibrium contribution is small relative to the equilibrium noise. We have shown (Fig. 2) for a Luttinger liquid wire connected to 1D noninteracting Fermi-gas leads, that while the relative contribution of the nonequilibrium thermal noise increases (relative to the equilibrium noise $P_{\rm JN}$) for a system with stronger interaction (see inset in Fig. 2), the total noise power is dominated by the equilibrium thermal noise even for a large temperature difference between the heat reservoirs (leads).

ACKNOWLEDGMENTS

We gratefully acknowledge support of this work from the National Research Council through the Twinning Program 1999–2000. I.V.K. thanks the School of Physics at the Georgia Institute of Technology for its hospitality. The research of E.N.B., A.G.S., and U.L. was also supported by the U.S. Department of Energy, Grant No. FG05-86ER45234.

[1] M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. L. Sohn *et al.* (Kluwer Academic Publishers, Dordrecht, 1997), p. 225.
[2] Ya. M. Blanter and M. Buttiker, Phys. Rep. **336**, 1 (2000).
[3] E. N. Bogachek, I. O. Kulik, and A. G. Shkorbatov, Fiz. Nizk. Temp. **11**, 1189 (1985) [Sov. J. Low Temp. Phys. **11**, 656 (1985)]; J. Phys.: Condens. Matter **3**, 8877 (1991).
[4] E. N. Bogachek and A. G. Shkorbatov, Fiz. Nizk. Temp. **11**, 643 (1985) [Sov. J. Low Temp. Phys. **11**, 353 (1985)].
[5] U. Sivan and Y. Imry, Phys. Rev. B **33**, 551 (1986).
[6] R. Landauer, Philos. Mag. **21**, 863 (1970); M. Buttiker, Phys. Rev. Lett. **57**, 1761 (1986).
[7] C. R. Proetto, Solid State Commun. **80**, 909 (1991).
[8] H. van Houten, L. W. Molenkamp, C. W. J. Beenakker, and C. T. Foxon, Semicond. Sci. Technol. **7**, B215 (1992).
[9] E. N. Bogachek, A. G. Scherbakov, and U. Landman, Solid State Commun. **108**, 851 (1998); Phys. Rev. B **60**, 11 678 (1999).
[10] D. E. Angelescu, M. C. Cross, and M. L. Roukes, Superlattices Microstruct. **23**, 673 (1998).
[11] L. G. C. Rego and G. Kirczenow, Phys. Rev. Lett. **81**, 232 (1998).
[12] M. P. Blencowe, Phys. Rev. B **59**, 4992 (1999).
[13] R. Fazio, F. W. J. Hekking, and D. E. Khmelnitskii, Phys. Rev. Lett. **80**, 5611 (1998).
[14] I. V. Krive, Fiz. Nizk. Temp. **24**, 498 (1998) [Low Temp. Phys. **24**, 377 (1998)]; Phys. Rev. B **59**, 12 338 (1999).
[15] K. Schwab, E. A. Henriksen, J. M. Worlock, and M. L. Roukes, Nature (London) **404**, 974 (2000).
[16] M. Buttiker, Phys. Rev. B **46**, 12 485 (1992).
[17] Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, 1996).
[18] L. G. C. Rego and G. Kirczenow, Phys. Rev. B **59**, 13 080 (1999).
[19] I. V. Krive and E. R. Mucciolo, Phys. Rev. B **60**, 1429 (1999).
[20] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **72**, 724 (1994).
[21] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **76**, 3192 (1996).
[22] I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (W. A. Benjamin, New York, 1965), Chap. 23.