PHYS 6107. Spring 2002.

HOMEWORK #4 $\sum = 100\ pts.$

<u>DUE:</u> Apr. 16 by 6 p.m. (in Dr. Marchenkov's mail box)

This homework deals with Bose-Einstein and Fermi-Dirac systems. There are no bonus problems in this assignment.

Problem I.1 (15 pts)

n-dimensional universe

In our three-dimensional universe, the following are well-known results from statistical mechanics and thermodynamics:

- The energy density of black-body radiation depends on the temperature as T^{α}, where $\alpha = 4$.
- In the Debye model of a solid, the specific heat at low temperatures depends on the temperature as T^{β}, where $\beta = 3$.
- The ratio of the specific heat at constant pressure to the specific heat at constant volume for a monatomic ideal gas is $\gamma = \frac{5}{3}$.

Derive the analogous results (i.e. what are α, β, γ) in the universe with n dimensions.

Problem I.2 (20 pts)

Consider a three-dimensional gas of bosons for which the single-particle energy is given by

$$\varepsilon_{\vec{p},n} = \frac{|\vec{p}|^2}{2m} + \alpha n,$$

where α is a positive constant and $n = -j, \ldots, j$ is an integer.

- Find expressions, valid in the thermodynamic limit, for the pressure P and the mean number of particles per unit volume N/V in terms of the temperature and the fugacity.
- Write down the condition for Bose-Einstein condensation for this system (find T_c).

Problem I.3 (25 pts)

- For a gas of free electrons in n dimensions, compute the isothermal compressibility $\kappa_T(0)$ at zero temperature, in terms of the mean number of particles per unit volume and the Fermi energy, ε_F.
- Estimate the Fermi temperature for metallic copper by treating the electrons as a gas of free particles. The density of copper is 8920 kg m^{-3}, its atomic weight is 63.5 and it may be assumed that there is one conduction electron per atom.
- Demonstrate, why is it correct, for all practical purposes, to treat electrons in copper as a degenerate Fermi gas?

Problem I.4 (20 pts)

Consider a gas of N electrons contained in a box of volume V, whose walls have N_0 absorbent sites, each of which can absorb one electron. Let $-\varepsilon_0$ be the energy of an electron absorbed at one of these sites and $\frac{|\vec{p}|^2}{2m}$ the energy of a free electron.

- For $N > N_0$, find the limits as $T \to 0$ and $T \to \infty$ of the number N_a of absorbed electrons and N_f of free electrons.
- For $N = N_0$, find the chemical potential $\mu(T)$ and the particle numbers $N_a(T)$ and $N_f(T)$ at low temperatures.

Problem I.5 (20 pts)

Consider a system of N non-interacting electrons/cm^3, each of which can occupy either a bound state with energy $\varepsilon = -E_d$ or a free-particle continuum with $\varepsilon = \dfrac{p^2}{2m}$. (This can be a semiconductor like Si with N shallow donors/cm^3)

- Compute the density of states as a function of ε in the continuum.
- Find an expression for the chemical potential in the low temperature limit.
- Compute the number of free electrons (i.e. electrons in the continuum) as a function of temperature in the low-temperature limit.