PHYS 6107. Spring 2002.

HOMEWORK #2 $\sum = 100\ pts. + 25\ bonus\ pts.$

<u>DUE:</u> Mar. 19 by 6 p.m. (in Dr. Marchenkov's mail box)

The scope of this homework covers "thermodynamics" topics which were discussed in class, including the work, phase equilibrium, and Clausius-Clayperon equation. Two "bonus" problems are not required, but they are rather funny and enlightening.

The second law of thermodynamics says basically the same thing as Murphy's law: in any closed system, disorder, or entropy, always increases with time. In other words, things always tend to go wrong.

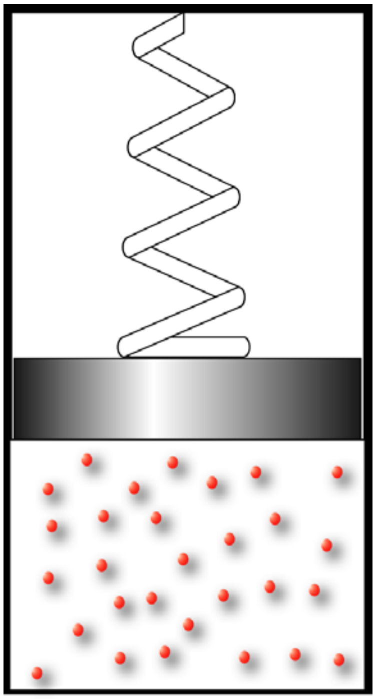
Stephen Hawking, "A Brief History of Time"

Thermodynamics is a funny subject. The first time you go through the subject, you don't understand it at all. The second time you go through it, you think you understand it - except for one or two small points. The third time you go through it, you know you don't understand it, but by that time you are so used to the subject that it doesn't bother you any more.

Arnold Sommerfeld

I. Thermodynamics (100pts)

Problem I.1 (15 pts)



One part of a cylinder is filled with one mole of a monatomic ideal gas ($pV = NT$) at a pressure 1 atm and temperature 300 K. A massless piston separates the gas from the other section of the cylinder which is evacuated but has a spring at equilibrium extension attached to it and to the opposite wall of the cylinder. The cylinder is thermally insulated from the rest of world, and the piston is fixed to the cylinder initially and then released. After reaching equilibrium, the volume occupied the gas is double the original. Neglecting the thermal capacities of the cylinder, piston, and spring, find the temperature and pressure of the gas.

Problem I.2 (15 pts)

A vessel with the volume of $V_1 = 3\,l$ contains $N_1 = 0.5\,mol$ of oxygen. Another vessel with the volume of $V_2 = 2\,l$ contains $N_2 = 0.5\,mol$ of nitrogen. Both gases are at room temperature. Find the maximal work, which can be obtained by mixing those gases in the vessel with the volume of $V_1 + V_2$ isothermally and adiabatically. Consider both gases ideal.

Problem I.3 (15 pts)

Helium gas, initially pumped into a vessel with the volume of $V = 20\,l$ at the pressure of 1 atm, diffused outside. Find the increase in the entropy of the helium and calculate the minimal work which should be applied to gather this amount of helium into the vessel from the atmosphere. Assume that in the atmosphere there is 1 atom of helium per 10^7 of other molecules.

Problem I.4 (20 pts)

For an elastic rubber band of length L, at temperature T and under a tension J, it is found experimentally that

$$g(T,L) \equiv \left(\frac{\partial J}{\partial T}\right)_L = \frac{aL}{L_0}\left[1-\left(\frac{L_0}{L}\right)^3\right]$$

$$f(T,L) \equiv \left(\frac{\partial J}{\partial L}\right)_T = \frac{aT}{L_0}\left[1+2\left(\frac{L_0}{L}\right)^3\right]$$

where L_0 is the length of the unstretched band (independent of temperature) and a is a constant.

- Obtain the equation of state of this system in the form $J = J(T,L)$.
- Assume that the heat capacity at constant length of the band is a constant C_L. If the band is stretched, adiabatically and reversibly, from L_0 at an initial temperature T_i to a final length L_f, what is its final temperature T_f?
- The band is now released, so it contracts freely to its natural length L_0. If no heat is exchanged with its surroundings during this contraction, find the changes in its temperature and entropy.

Problem I.5 (20 pts)

The curve separating the liquid and gas phases ends in the critical point $\left(V_c, T_c\right)$ where $\left(\frac{\partial P}{\partial V}\right)_{T=T_c}=0$. Using arguments based on thermodynamic stability, find

$$\left(\frac{\partial^2 P}{\partial V^2}\right)_{T=T_c}$$

at the critical point.

Problem I.6 (15 pts)

Helium-3 (^3He) at low pressures can remain in the liquid phase down to the absolute zero $T=0\ K$. The lowest solidification pressure is 28.9 atm $\left(T \neq 0\right)$. The molar entropy in the liquid phase at low temperatures can be expressed as

$$S=\frac{RT}{0.22\left[K\right]}.$$

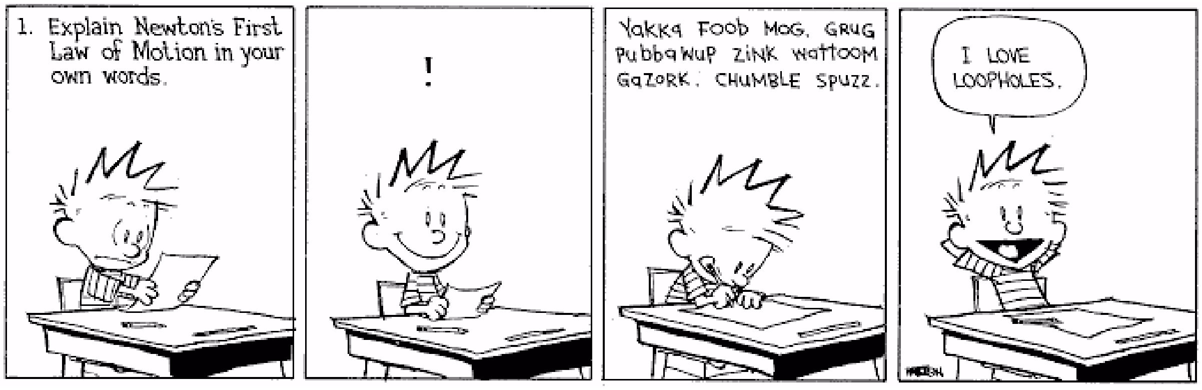
The molar entropy of the solid phase is temperature-independent and equal to $S=R\ln 2$. The molar volume difference in the liquid and the solid phase is

$$\Delta V=V_{liquid}-V_{solid}\approx 1.25\frac{cm^3}{mol}.$$

Using this information, calculate

- the temperature $T_{\min}$ corresponding to the minimum in melting pressure;
- the temperature dependence of the melting latent heat (sketch it);
- the solidification pressure at $T=0$.

II. Bonus Problems (25pts)



Problem II.1 (10 pts)

Why Bother?

A physicist and an engineer find themselves in a mountain lodge where the only heat is provided by a large woodstove. The physicist argues that they cannot increase the total energy of the molecules in the cabin, and therefore it makes no sense to continue putting logs into the stove. The engineer strongly disagrees, referring to the laws of thermodynamics and common sense. Who is right? Why do we keep heating the room?

Problem II.2 (15 pts)

Compare (give a numerical estimate) the decrease in the entropy of a reader's brain during the reading of a book with the increase in entropy due to illumination (by means of an electric light bulb).