As in the problem of classical gas adsorption, we'll apply the grand canonical ensemble. The chemical potential is determined by the constraint $N_a(T,\mu) + N_f(T,\mu) = N$.

The average number of electrons absorbed at each site is $(1+e^{-\beta(\mu+\varepsilon_0)})^{-1}$, so the average number of absorbed electrons is

$$N_a = \frac{N_0}{1+e^{-\beta(\mu+\varepsilon_0)}}$$

The number of free electrons is (see e.g. previous problem):

$$N_f = \frac{4}{\pi^2}\left(\frac{m}{2\hbar^2}\right)^{3/2}\int_0^\infty d\varepsilon \, \frac{\sqrt{\varepsilon}}{e^{\beta(\varepsilon-\mu)}+1} =$$

$$= \frac{4}{\pi^2}\left(\frac{m}{2\hbar^2}\right)^{3/2} V T^{3/2} I_{1/2}\left(\frac{\mu}{T}\right)$$

(a) If $N > N_0$, then N_f is always finite and non-zero. This requires $z \to \infty$ when $T \to 0$ and $z \to 0$ when $T \to \infty$ (z = fugacity). In the low-temperature limit, we find $N_a \to N_0$ and $N_f \to N - N_0$ while, in the high temperature limit, $N_a \to 0$, $N_f \to N$

(b) For $N = N_0$, the constraint $N = N_a + N_f$ becomes $N_0 - N_a = N_f$, or, explicitly

$$\frac{N_0/V}{e^{(\mu+\varepsilon_0)/T}+1} = \frac{4}{\pi^2}\left(\frac{m}{2\hbar^2}\right)^{3/2} T^{3/2} I_{1/2}\left(\frac{\mu}{T}\right)$$

Let's determine behavior of μ/T as $T \to 0$

Suppose $\frac{\mu}{T} \to$ finite value; then the left-hand side varies as $e^{-\frac{\varepsilon_0}{T}}$ and the right-hand side varies as $T^{3/2}$ which is inconsistent.

Suppose $\frac{\mu}{T} \to +\infty$; LHS behaves as $e^{-(\mu+\varepsilon_0)/T}$ and RHS $I_{1/2}\left(\frac{\mu}{T}\right) \sim \left(\frac{\mu}{T}\right)^{3/2} \Rightarrow$ also inconsistent.

The consistent behavior is $\underline{\frac{\mu}{T} \to -\infty \text{ and } z \to 0.}$

$$\therefore \quad I_{1/2}\left(\frac{\mu}{T}\right) = z \int_0^\infty dx\, x^{1/2} e^{-x} (1 + O(z)) = \frac{\sqrt{\pi}}{2} z + \ldots$$

$$\Downarrow$$

$$\frac{N_0/V}{e^{(\mu+\varepsilon_0)/T} + 1} \approx \frac{\sqrt{\pi}}{2} \frac{4}{\pi^2} \left(\frac{m}{2\hbar^2}\right)^{3/2} T^{3/2} z$$

$$\Downarrow$$

$$e^{\frac{\varepsilon_0}{T}} z^2 + z - \underbrace{\text{const}}_{\frac{2N_0}{\sqrt{\pi} V \left(\frac{4}{\pi^2}\right)\left(\frac{m}{2\hbar^2}\right)^{3/2}}} T^{-3/2} = 0$$

when $T \to 0$

$$z \approx \sqrt{\text{const}}\; T^{-3/4} e^{-\frac{\varepsilon_0}{2T}}$$

$$\Downarrow$$

$$\mu \approx \frac{1}{2}\left[T \ln\left(\frac{\text{const}}{T^{3/2}}\right) - \varepsilon_0\right]$$

$$\Downarrow$$

$$N_f \approx \left(\frac{\sqrt{\pi}}{2} \cdot \frac{4}{\pi^2} \left(\frac{m}{2\hbar^2}\right)^{3/2} N_0 V\right)^{1/2} T^{3/4} e^{-\frac{\varepsilon_0}{2T}}$$

and, of course, $N_a = N_0 - N_f$