(a) An ideal Fermi gas is most conveniently treated using the grand canonical ensemble:

$$\Xi = \prod_{\bar{p},s} \sum_{n_{\bar{p},s}=0}^{1} e^{-\beta(\varepsilon_{\bar{p},s}-\mu)\, n_{\bar{p},s}} = \prod_{\bar{p},s} \left(1 - e^{\beta(\varepsilon_{\bar{p},s}-\mu)}\right)$$

The index $s = \pm\frac{1}{2}$ labels spin states. If the single-particle energy is independent of spin (and we shall take $\varepsilon = \frac{|\bar{p}|^2}{2m}$), then for a gas in a container of macroscopic size the pressure and number density can be written as

$$p = \frac{T}{V} \ln \Xi = \frac{T}{V} \int_0^\infty d\varepsilon\, g(\varepsilon) \ln\left(1 + e^{-\beta(\varepsilon-\mu)}\right)$$

$$\langle n \rangle = \frac{\langle N \rangle}{V} = \frac{1}{V} \left( \frac{\partial \ln \Xi}{\partial(\beta\mu)} \right)_{\beta,V} = \frac{1}{V} \int_0^\infty d\varepsilon\, g(\varepsilon) \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

Recall now (or find in literature) that in $n$ dimensions the density of states for free particles is $g(\varepsilon) = \text{const} \cdot V \cdot \varepsilon^{\frac{n}{2}-1}$.
At zero temperature, the occupation numbers are $n(\varepsilon) = \left(1 + e^{\beta(\varepsilon-\mu)}\right)^{-1} = \Theta(\varepsilon_F - \varepsilon)$ step function

$$\therefore \quad \langle n \rangle = \frac{\text{const}}{n/2} \varepsilon_F^{\frac{n}{2}}$$

integration by parts

$$p = \frac{2}{n} \frac{1}{V} \int_0^\infty d\varepsilon\, g(\varepsilon)\, \varepsilon\, n(\varepsilon) = \frac{\text{const}}{\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)} \varepsilon^{\frac{n}{2}+1} =$$

$$= \frac{\langle n \rangle \varepsilon_F}{\frac{n}{2}+1}$$

The compressibility can be found from

$$k_T(0)^{-1} = -V\left(\frac{\partial P}{\partial V}\right)_{N,T=0} = \langle n\rangle \frac{dP}{d\langle n\rangle} = \frac{2}{n}\left(\frac{n}{2}+1\right)P =$$
$$= \frac{2}{n}\,\varepsilon_F \langle n\rangle$$

or $\quad k_T(0) = \dfrac{n}{2\langle n\rangle \varepsilon_F}$.

In this calculation we have used two previous equations to write

$$P = \frac{\text{const}}{\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)}\left(\frac{n}{2\,\text{const}}\langle n\rangle\right)^{\frac{2}{n}\left(\frac{n}{2}+1\right)}$$

Alternatively, this result can be obtained by making use of the relation $\left(\frac{\partial \mu}{\partial N}\right)_{T,V} = \frac{V}{N^2 K_T}$

$$\Downarrow$$

$$k_T(0) = \frac{V}{N^2}\left(\frac{\partial N}{\partial \mu}\right)_{V,T=0} = \frac{1}{\langle n\rangle^2}\frac{d\langle n\rangle}{d\varepsilon_F} = \frac{n}{2\langle n\rangle \varepsilon_F}$$

(b) In 3D, the const is

$$\text{const} = \underbrace{\frac{4}{\pi^2}}_{\text{factor 2 for spin degeneracy is included}}\left(\frac{m}{2\hbar^2}\right)^{3/2}$$

We then find that

$$T_F = \varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2\langle n\rangle\right)^{2/3}$$

$m = 9.11\cdot 10^{-31}$ kg $\quad N_e \simeq N_{atoms} = \langle n\rangle \simeq \frac{10^3 \rho}{M} N_A \simeq$
$\simeq 8.5\cdot 10^{28}\ m^{-3}$ ($\rho$ - density $M = 63.5$) $\Rightarrow$ $\underline{\underline{T_F \simeq 82000\,K}}$

(c) $T_F \gg 300K$