The quantization of a particle in a 1D square well gives the following energy spectrum:

$$E(n) = \frac{n^2 \hbar^2 \pi^2}{2mL^2} \qquad n = 1, 2, 3, \ldots$$

and so the canonical partition function has the form:

$$Z(T,L) = \sum_{n=1}^{\infty} e^{-\frac{n^2\Theta}{T}}, \quad \Theta = \frac{\hbar^2\pi^2}{2mL^2}$$

At low temperatures, express $Z(T,L)$ in terms of the variable $x = e^{-\Theta/T} = e^{-\beta\Theta}$.

Since x is small when $T \ll \Theta$, we keep only few terms of the power series expansion:

$$Z(T,L) = x(1 + x^3 + x^8 + x^{15} + \ldots)$$

Derivatives with respect to β and L can be expressed as:

$$\frac{\partial}{\partial \beta} = -\Theta x \frac{\partial}{\partial x} \qquad \frac{\partial}{\partial L} = \frac{2\Theta}{LT} x \frac{\partial}{\partial x}$$

$$\therefore \quad \langle E \rangle = -\frac{\partial}{\partial \beta} \ln Z = \Theta x \frac{\partial}{\partial x} \ln Z = \Theta(1 + 3x^3 + \ldots)$$

Then

$$c_L = -\beta^2 \frac{\partial \langle E \rangle}{\partial \beta} = 9\left(\frac{\Theta}{T}\right)^2 (x^3 + \ldots) \approx \underline{\underline{9\left(\frac{\Theta}{T}\right)^2 e^{-\frac{3\Theta}{T}}}}$$

To obtain the equation of state, we need only calculate the 1D pressure as a function of T and L. We find that

$$P = -\frac{\partial}{\partial L} F(T,L) = T \frac{\partial \ln Z}{\partial L} = \frac{2T}{L}\left(\frac{\Theta}{T}\right)(1 + 3x^3 + \ldots)$$

__Note__ a simple relation $P = \frac{2}{L}\langle E \rangle$.

At high temperatures, $T \gg \Theta$, we substitute, as usual, the sum by an integral.

Using $p = \frac{n\pi\hbar}{L}$ and $dn = \frac{L}{\pi\hbar}\,dp$:

$$Z(T,L) = \frac{L}{\pi\hbar}\int_0^\infty e^{-\frac{p^2}{2mT}}\,dp = L\left(\frac{mT}{2\pi\hbar^2}\right)^{1/2} = \frac{1}{\sqrt{\beta}}L\left(\frac{m}{2\pi\hbar^2}\right)^{1/2}$$

$$\therefore \quad \langle E\rangle = -\frac{\partial}{\partial\beta}\ln Z = \frac{1}{2}T$$

$$C_L = \left(\frac{\partial\langle E\rangle}{\partial T}\right)_L = \frac{1}{2}$$

$$P = \left(\frac{\partial}{\partial L}(T\ln Z)\right)_T = \frac{T}{L}$$

As might have been expected, since these are precisely the results for a one-dimensional classical gas.