HW #2 | 1.1

FOR a SYSTEM WITH NO HEAT TRANSFER, THE ENERGY CHANGE IS:

$$\Delta E = \int_0^1 P\, dV$$

, where $\{0,1\}$ correspond to the initial and final equilibrium states of the SYSTEM:

$\{T_0, P_0, V_0\}$ and $\{T_1, P_1, V_1\}$

In this case, the gas is expanding, therefore some POSITIVE work is done by the gas, which indicates that $\Delta E < 0$ and T decreases.

FOR an ideal gas: $\Delta E = C_v \Delta T = C_v (T_1 - T_0)$.

THE WORK DONE BY GAS goes into compressing the spring: $U = \frac{Kx^2}{2} = -\Delta E$ (*), where x is the change in the PISTON POSITION.

On the other hand, when equilibrium is reached:

$$F = Kx = P_1 \cdot A,$$

where A is the AREA of the piston.

T in degrees

$$\therefore \quad K = \frac{P_1 A}{x} \underset{\text{ideal gas 1 mol}}{=} \frac{R T_1 A}{x V_1} \qquad (**)$$

$$(**) \to (*) \Rightarrow U = \frac{R T_1 A x}{2 V_1}$$

Notice that: $A \cdot x = V_1 - V_0 = V_0$

$$\therefore \quad U = \frac{R T_1}{4} \Rightarrow$$

$$\therefore \quad \Delta E = C_v (T_1 - T_0) = -\frac{R T_1}{4}$$

and

$$T_1 = \frac{c_v T_0}{c_v + \frac{1}{4}R} = \frac{T_0}{1 + \frac{R}{4c_v}} = \frac{300}{1 + \frac{1}{6}} \approx \underline{\underline{257\,K}}$$

$$c_v = \frac{3}{2}R$$

AS FOR THE PRESSURE:

$$p_0 V_0 = R T_0,$$

$$V_1 = 2V_0 \Rightarrow \quad p_1 V_1 = 2 p_1 V_0 = R T_1$$

$$\Downarrow$$

$$p_1 = \frac{1}{2}\frac{T_1}{T_0} p_0 = \frac{3}{7} p_0 \approx \underline{\underline{0.43 \text{ atm}}}$$