

Problem III.5

ZIPPER

① A partition function:

$$Z = \sum_{n=0}^N e^{-\frac{n\epsilon}{T}}$$

since a state with n open links has an energy $\epsilon_n = n\epsilon$.

So:

$$Z = \sum_{n=0}^N \eta^n = \frac{1-\eta^{N+1}}{1-\eta}, \quad \eta = e^{-\frac{\epsilon}{T}}$$

② The average number of open links $\langle n \rangle$ is given by:

$$\langle n \rangle = \frac{1}{Z} \sum_{n=0}^N n \eta^n = \frac{\sum_{n=0}^N n \eta^n}{\sum_{n=0}^N \eta^n} =$$

$$= \eta \frac{\partial}{\partial \eta} \ln Z = \eta \frac{\partial}{\partial \eta} \ln \left(\frac{1-\eta^{N+1}}{1-\eta} \right) =$$

$$= \frac{\eta}{1-\eta} - \frac{(N+1)\eta^{N+1}}{1-\eta^{N+1}}$$

If $T \ll \epsilon$ then $\eta \ll 1$ and

$$\langle n \rangle \approx \eta = e^{-\frac{\epsilon}{T}}$$

which does not depend on N .

It is also zipped up tight!