Problem III.1

Let us start from the GIBBS factor

$$w_{N,j}(V,T,\mu) = \frac{\exp(-(E_{Nj}(N,V) - \mu N)/T)}{\Xi(V,T,\mu)}$$

$$\Xi = \sum_{N,j} \exp\left(-(E_{Nj}(N,V) - \mu N)/T\right)$$

grand partition function

where N,j specify the j^{th} quantum state of the system when it contains N particles.

Now for a system of identical particles:

$$E = \sum_k n_k \varepsilon_k$$

$$N = \sum_k n_k$$

and a given state of the system is specified by the occupation of the single-particle states k:

$$\{n_k\} = (n_1, n_2, \dots)$$

So in this case the GPF is expressed as

$$\Xi(V,T,\mu) = \sum_{\{n_k\}} \exp\left(-\sum_k \frac{(\varepsilon_k - \mu) n_k}{T}\right) =$$

$$= \sum_{\{n_k\}} \prod_k \exp\left(-\frac{(\varepsilon_k - \mu) n_k}{T}\right)$$

Now the n_k are all <u>independent</u>, so that

$$\Xi(V,T,\mu) = \prod_k \sum_{\{n_k\}} \exp\left(-\frac{(\varepsilon_k - \mu) n_k}{T}\right)$$

Problem III.1

Now we have the definition:

$$\Xi_k(V,T,\mu) = \sum_{n_k} \exp\left(-\frac{(\varepsilon_k-\mu)n_k}{T}\right),$$

so that the GPF may be expressed as the product :

$$\Xi(V,T,\mu) = \prod_k \Xi_k(V,T,\mu)$$

as required.

<u>It is important to note the logic of this answer</u>. It is not good enough to start from Ξ_k and then to argue that it <u>must</u> be ~~distinguished~~ multiplicative.