## Problem II.1

The change in entropy, when particles flow is given by

$$\Delta S = \left( \frac{\partial S_1}{\partial N} - \frac{\partial S_2}{\partial N} \right) \Delta N_1 \geq 0$$

But since

$$\frac{\partial S}{\partial N} = -\frac{\mu}{T}$$

then $\Delta S = \frac{1}{T} (\mu_2 - \mu_1) \Delta N_1 \geq 0$

considering temperature to be the same in both systems.

We conclude that when $\mu_2 > \mu_1$, then $\Delta N_1$ will be positive $\Rightarrow$ $N_1$ will increase.

$\therefore$ particles will flow from <u>high to low $\mu$.</u>

## Problem III.2

Average speed:

$$\langle v \rangle \equiv \int_{v=0}^{\infty} v\, f(v)\, dv = A \int_0^{\infty} v^2 e^{-\frac{v}{v_0}}\, dv =$$

$$= A \left[ e^{-\frac{v}{v_0}} \left(-2v_0^3 - 2v_0^2 v - v_0 v^2\right) \right]_0^{\infty} =$$

$$= 2A\, v_0^3$$

Now : $\int_0^{\infty} f(v)\, dv = 1 \Rightarrow \left[ A e^{-\frac{v}{v_0}} \left(-v v_0 - v_0^2\right) \right]_0^{\infty} = 1$

$\nearrow$ normalization condition

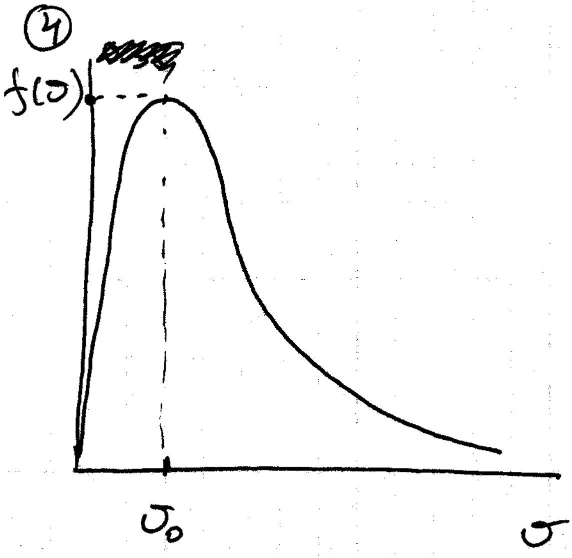
① $\boxed{\therefore \quad A = \frac{1}{v_0^2}}$

② $\boxed{\langle v \rangle = 2v_0}$

③ Most probable speed corresponds to the biggest probability, i.e. it is obtained from $\frac{d}{dv} f(v) = 0$, which, in principle, is different from $\langle v \rangle$. In our case

$v_{mp} = v_0$

④



**Problem II.3** | NOTE: Several solutions are possible.

The time intervals of such a system being in the homogeneous and separated states ($t_0$ and $t_1$) have the same ratio as corresponding probabilities.

$$\frac{t_0}{t_1} = \frac{w_0}{w_1} = \exp(S_0 - S_1) \quad , \text{since } w \propto e^{S}$$

On the other hand: $S_0 - S_1 = \ln 2^{2N}$

$$\Downarrow$$

$$\frac{t_0}{t_1} = 2^{2N}$$

The time $t_1$ is the time for a separated state to be destroyed. We can estimate it usin $t_1 \sim \frac{l}{\langle v \rangle}$, where $l \sim 10\,\text{cm}$ is a typical size of the vessel, and $\langle v \rangle \sim 10^3 \frac{m}{s}$ typical molecular velocity at room temperature. $t_0 \sim 10^{10}$ years $\sim 10^{17}$ s.

$$\therefore \quad N \approx \frac{\log_x \frac{t_0}{t_1}}{2 \log_x 2} \approx 35.$$

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This is one of the possible solutions!