

$$Y \psi'''' = p(x) \quad ; \quad 0 \leq x \leq L$$

$$\psi(0) = 0 \quad ; \quad \psi'(0) = 2\psi'(L) \quad ; \quad \psi(L) = a \quad ; \quad \psi''(0) = 0$$

Multiply through by $G(x)$ and integrate:

$$\int_0^L G(x) \psi''''(x) dx = Y^{-1} \int_0^L G(x) p(x) dx \quad [\star]$$

(a) On the left hand side, integrate by parts four times to get

$$[G \psi'''' - G' \psi''' + G'' \psi'' - G''' \psi] \Big|_0^L + \int_0^L G''''(x) \psi(x) dx$$

The integral reduces to $\psi(\xi)$ if we choose

$$G'''' = \delta(x - \xi)$$

Choose the boundary conditions on G so as to kill the boundary terms involving information about ψ that we don't know. Three of these we get instantly:

$$G(0) = 0 \quad \text{since we don't know } \psi''''(0)$$

$$G(L) = 0 \quad \text{since we don't know } \psi''''(L)$$

$$G'(L) = 0 \quad \text{since we don't know } \psi'''(L)$$

The fourth boundary condition requires a little more thought. We have to evaluate

$$\begin{aligned} G''(L) \psi'(L) - G''(0) \psi'(0) &= G''(L) \psi'(L) - G''(0) 2\psi'(L) \\ &= [G''(L) - 2G''(0)] \psi'(L) \end{aligned}$$

which means we should choose (since we don't know $\psi'(L)$)

$$G''(L) = 2G''(0)$$

(b) Using Eq.(1) and the four boundary conditions on G , Eq. [★] reduces to

$$G(\xi) = Y^{-1} \int_0^L G(x) p(x) dx + a G'''(L)$$

or, upon renaming the dummy variable and switching $\xi \rightarrow x$:

$$G(x) = Y^{-1} \int_0^L G(s) p(s) ds + a G'''(L)$$