

$$x^2 \Psi'' + 3x\Psi' + (Ax^2 - B)\Psi = 0 \quad ; \quad x > 0$$

$$P_0(x) = x^2 \quad ; \quad P_1(x) = 3x \quad ; \quad P_2(x) = Ax^2 - B$$

(a) Clearly,  $P_0' = 2x \neq P_1$ , so the equation is not self-adjoint.

(b) The “integrating factor” is

$$\begin{aligned} f(x) &= \exp \left\{ \int^x \frac{P_1 - P_0'}{P_0} dx' \right\} \\ &= \exp \{ \log x \} \\ &= x \end{aligned}$$

so multiply the B-W equation through by  $x$ :

$$x^3 \Psi'' + 3x^2 \Psi' + (Ax^3 - Bx)\Psi = 0$$

which is equivalent to the manifestly self-adjoint

$$(x^3 \Psi')' + Ax^3 \Psi - Bx \Psi = 0$$

(c) Treating  $A$  as the eigenvalue, group terms this way:

$$\left\{ (x^3 \Psi')' - Bx \Psi \right\} + Ax^3 \Psi = 0$$

which yields the self-adjoint S-L eigenvalue problem

$$\mathcal{L}\Psi_n + \lambda_n w(x)\Psi_n = 0$$

with identifications

$$\mathcal{L} = (x^3 \Psi')' - Bx \Psi \quad ; \quad \lambda_n = A \quad ; \quad w(x) = x^3$$