

(a) The given mass density is

$$\rho(\vec{r}) = A \delta(r - R) \delta(\cos \theta)$$

Integrate over all space to get the total mass  $M$

$$\begin{aligned} M &= \int \int \int \rho(\vec{r}) d\tau \\ &= \int \int \int A \delta(r - R) \delta(\cos \theta) r^2 \sin \theta dr d\theta d\phi \\ &= A \int_{r=0}^{\infty} \delta(r - R) r^2 dr \int_{\theta=0}^{\pi} \delta(\cos \theta) \sin \theta d\theta \int_{\phi=0}^{2\pi} d\phi \\ &= A \cdot R^2 \cdot 1 \cdot 2\pi \\ &= 2\pi AR^2 \quad \Rightarrow \quad \boxed{A = \frac{M}{2\pi R^2}} \end{aligned}$$

(b) The given charge density is

$$\rho(\vec{r}) = \frac{B}{r^2} \delta(\cos \theta - 1)$$

for  $r < L$  and is zero otherwise. Integrate over all space to get the total charge  $Q$

$$\begin{aligned} Q &= \int \int \int \frac{B}{r^2} \delta(\cos \theta - 1) r^2 \sin \theta dr d\theta d\phi \\ &= B \int_{r=0}^L dr \int_{\theta=0}^{\pi} \delta(\cos \theta - 1) \sin \theta d\theta \int_{\phi=0}^{2\pi} d\phi \\ &= B \cdot L \cdot 1 \cdot 2\pi \\ &= 2\pi BL \quad \Rightarrow \quad \boxed{B = \frac{Q}{2\pi L}} \end{aligned}$$