$$\Psi(x) = \begin{cases} A\,e^{ikx} + B\,e^{-ikx} & ; \quad \text{for} \quad x < 0 \\ C\,e^{ikx} & ; \quad \text{for} \quad x > 0 \end{cases}$$

Continuity of Ψ at $x = 0$ implies

$$A + B = C \tag{1}$$

Integrate the differential equation over a small neighborhood $x \in (-\epsilon, \epsilon)$:

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{\epsilon} \Psi''(x)\,dx + \alpha\int_{-\epsilon}^{\epsilon} \delta(x)\,\Psi(x) = E\int_{-\epsilon}^{\epsilon} \Psi(x)\,dx$$

$$-\frac{\hbar^2}{2m}\left[\Psi'(\epsilon) - \Psi'(-\epsilon)\right] + \alpha\,\Psi(0) = E\int_{-\epsilon}^{\epsilon} \Psi(x)\,dx$$

In the limit $\epsilon \to 0$, the integral on the righthand side goes to zero (since Ψ is bounded), so that

$$-\frac{\hbar^2}{2m}\left[\Psi'(0^+) - \Psi'(0^-)\right] + \alpha\,\Psi(0) = 0$$

Evaluate $\Psi'(0^\pm)$ and $\Psi(0)$ from the expression at the top of the page:

$$\Psi'(0^+) = ikC \quad ; \quad \Psi'(0^-) = ik(A - B) \quad ; \quad \Psi(0) = C$$

so that

$$-\frac{\hbar^2}{2m}\,ik\,(C - A + B) + \alpha\,C = 0 \tag{2}$$

Equations (1) and (2) can be solved for the two ratios B/A and C/A. After some algebra, I find

$$\frac{B}{A} = \frac{-\imath\beta}{1 + \imath\beta} \quad \text{and} \quad \frac{C}{A} = \frac{1}{1 + \imath\beta}$$

where $\beta = (m\alpha)/(k\hbar^2)$ and so

$$R = \frac{|B|^2}{|A|^2} = \frac{\beta^2}{1 + \beta^2} \quad \text{and} \quad T = \frac{|C|^2}{|A|^2} = \frac{1}{1 + \beta^2}$$