

Consider the Modified Bessel-Wiesenfeld equation, which arises in the as-yet-unappreciated Superduperstring Theory:

$$x^2 \Psi'' + 3x\Psi' + (Ax^2 - B)\Psi = 0 \quad ; \quad x > 0$$

- (a) Show (by checking the appropriate condition on the coefficients) that this equation is not formally self-adjoint.
- (b) Show that this equation can be put into self-adjoint form by multiplying through by the appropriate function $f(x)$, and determine $f(x)$.
- (c) Now suppose that B is a known constant, and we want to solve this equation subject to fixed boundary conditions. In that case solutions exist only for certain (discrete) values of A , so that A plays the role of the eigenvalue. Identify the *weight function* $w(x)$ by writing the problem in the standard form

$$\mathcal{L}\Psi_n + A_n w(x)\Psi_n = 0$$

where \mathcal{L} is the self-adjoint differential operator.