Quantum Tunnelling. Consider the one dimensional problem of a beam of particles (mass m and kinetic energy E) incident on a localized barrier. The corresponding Schrodinger equation is

$$-\frac{\hbar^2}{2m}\Psi'' + \alpha\,\delta(x)\,\Psi \quad = \quad E\,\Psi$$

where $\Psi(x)$ is the wavefunction, $\hbar$ is Planck's constant divided by 2π, α is a positive constant, and the primes denote differentiation with respect to x. Suppose the beam is incident from the left. Then, for $x \neq 0$, we have

$$\Psi(x) = \begin{cases} \Psi(x) \;=\; A\,e^{ikx} + B\,e^{-ikx} \;; & \text{for} \quad x < 0 \\ \Psi(x) \;=\; C\,e^{ikx} \;; & \text{for} \quad x > 0 \end{cases}$$

where the amplitudes A, B, and C are complex constants, and $k = \sqrt{2mE/\hbar^2}$.

By demanding that ψ is continuous, and integrating the Schrodinger equation over a vanishingly small interval around $x = 0$, determine the reflection coefficient R and the transmission coefficient T, given by

$$R = \frac{|B|^2}{|A|^2} \qquad \text{and} \qquad T = \frac{|C|^2}{|A|^2} \quad .$$

Physically, the transmission coefficient is the probability that a particle tunnels through the barrier. Of course, this is a purely quantum mechanical effect: classically, the particle cannot get through if E is less than the maximum value of the potential barrier (which is infinite for the δ-function).

As a check on your calculation, note that $R + T = 1$.