

1. Evaluate the integral:

$$\int_{-\infty}^{\infty} e^x \delta(3x^2 - 1) dx$$

Express your answer as a decimal number, accurate to 1%.

Hint: You'll want to use the general property

$$\delta(g(x)) = \sum_a \frac{\delta(x - a)}{|g'(a)|}$$

where $\{a\}$ are the (simple) zeroes of $g(x)$.

2. Consider the function

$$f(x) = e^{-|x|}$$

Show that

$$\frac{d^2 f}{dx^2} = f(x) + c \delta(x - x_0)$$

and explicitly determine the two constants, c and x_0 .

Note: If you want to take a numerical approach to this problem, you might try using

$$\lim_{\beta \rightarrow \infty} x \tanh(\beta x) = |x|$$

and plot the corresponding functions $f_\beta(x)$, $f'_\beta(x)$, $f''_\beta(x)$ for a sequence of increasing values of β .