(a) The displacement vector

$$\vec{r} = \hat{\mathbf{x}}\, a \cos\xi + \hat{\mathbf{y}}\, b \sin\xi$$

traces out an ellipse in x-y plane as the parameter ξ runs from zero to 2π. By integrating around the ellipse, explicitly evaluate the vector quantity

$$\oint \vec{\mathbf{r}} \times \mathbf{d}\vec{\mathbf{r}}$$

and verify that its magnitude is $2\pi ab$, i.e. twice the area of the ellipse.

(b) Now demonstrate, quite generally, that for any closed curve lying entirely in the x-y plane, the magnitude of $\oint \vec{\mathbf{r}} \times \mathbf{d}\vec{\mathbf{r}}$ is equal to twice the area of the enclosed curve. Do this by applying the following form of Stokes' theorem

$$\int_S (d\vec{\sigma} \times \nabla) \times \vec{\mathbf{P}} = \oint d\vec{\lambda} \times \vec{\mathbf{P}}$$

and making a suitable choice for $\vec{\mathbf{P}}$. *NOTE: since the loop lies entirely in the x-y plane, the area element $d\vec{\sigma}$ always has its normal oriented along the z-axis.*