

Consider the set of curvilinear coordinates (u, v, w) which are related to the usual Cartesian coordinates (x, y, z) by

$$x = \frac{1}{2} (u^2 - v^2)$$

$$y = uv$$

$$z = w$$

- (a) Check that (u, v, w) is an orthogonal coordinate system.
- (b) Determine the scale factors.
- (c) Express $\nabla^2 f$ in these non-Cartesian coordinates, where $f(u, v, w)$ is a scalar function.
- (d) Find the area of a patch lying on the surface $w = 3$ which runs over $u \in (0, 2)$ and $v \in (1, 2)$. Your answer should be a pure number.