Problem 1. Prove the identity

$$\varepsilon_{ijk}\,\varepsilon_{ipq} = \delta_{jp}\,\delta_{kq} - \delta_{jq}\,\delta_{kp}$$

where ε is the fully antisymmetric rank-3 tensor and δ is the Kronecker delta.

Problem 2. Until recently, I employed a super-enthusiastic postdoc by the name of Eager N. Diligent. One day, after having just learned the "epsilon i-j-k" notation, Dr. Diligent burst into my office saying, "Hey, Professor W., I've discovered a new vector identity which isn't in any of my textbooks! This tensor notation is absolutely amazing!!" Dr. Diligent's identity was

$$\nabla \times \left(\vec{A} \times \vec{B}\right) = \left(\nabla \cdot \vec{B}\right)\vec{A} - \left(\nabla \cdot \vec{A}\right)\vec{B}$$

Here is Diligent's proof. Your job is to edit it: identify any specific errors and correct the corresponding steps, to arrive at the true identity.

$$\begin{aligned}
\left\{\nabla \times \left(\vec{A} \times \vec{B}\right)\right\}_i &= \varepsilon_{ijk}\,\partial_j\,\varepsilon_{k\ell m}\,A_\ell\,B_m \\
&= \varepsilon_{ijk}\,\varepsilon_{k\ell m}\,\partial_j\,A_\ell\,B_m \\
&= \varepsilon_{kij}\,\varepsilon_{k\ell m}\,\partial_j\,A_\ell\,B_m \\
&= \left(\delta_{i\ell}\,\delta_{jm} - \delta_{im}\,\delta_{j\ell}\right)\partial_j\,A_\ell\,B_m \\
&= \partial_j\,A_i\,B_j - \partial_j\,A_j\,B_i \\
&= \partial_j\,B_j\,A_i - \partial_j\,A_j\,B_i \\
&= \left\{\left(\nabla \cdot \vec{B}\right)\vec{A} - \left(\nabla \cdot \vec{A}\right)\vec{B}\right\}_i \qquad \textit{q.e.d.}
\end{aligned}$$