Physics 3143 — Homework # 4

1. For the square well potential, start with the expression

$$A = \frac{F}{4} e^{ik_1 a}\left\{4\cos k_2 a - 2i\left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right)\sin k_2 a\right\}$$

to determine the expression for the transmission coefficient, T

$$T = |F|^2/|A|^2 = 1\Big/\left(1 + \tfrac{1}{4}\,\frac{V_0^2}{E(E+V_0)}\sin^2 k_2 a\right)$$

2. A beam of neutrons ($m_n = 1.675 \times 10^{-27}$ kg) is incident on a nucleus. Consecutive transmission maxima are observed for beam energies of 1.150, 23.656, and 50.254 MeV. Treating the nucleus as a one dimensional square-well potential:
   (a) What is the width of the potential? (ans. 10.0 fm)
   (b) What is the depth of the potential? (ans. 50.0 MeV)

The observations are related to the Ramsauer – Townsend Effect

3. Obtain an expression for B in terms of F similar to that above for the coefficient, A, of the incident flux (note class notes) and show that

$$|F|^2 + |B|^2 = |A|^2$$

4. Using $e^{i\phi} = \cos\phi + i\sin\phi$, show that $\sin i\phi = i\sinh\phi$ (recall $\sinh\phi = \frac{1}{2}(e^{\phi} - e^{-\phi})$, $\cosh\phi = \frac{1}{2}(e^{\phi} + e^{-\phi})$)

5. For a barrier of height 6.0 eV. and width 1.0 nm and an electron of energy 1.0 eV., show that the transmission coefficient changes by an order of magnitude for a change in width of 0.1 nm.

6. Show that the probability current density inside the barrier is given by $j_x = -ik_2\hbar/m\,(CD^* - C^*D)$ where

$$\psi_{II}(x) = Ce^{k_2 x} + De^{-k_2 x}$$

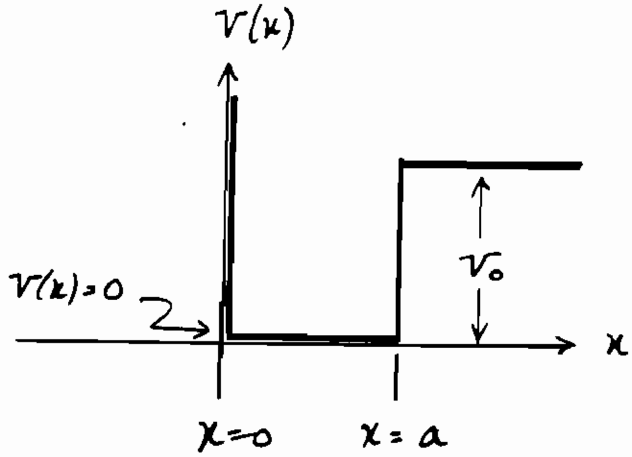
(Note that it is important that C and D are not real)

7. The deuterium nucleus (a bound state of a proton and a neutron) has one bound state. The force acting between a proton and a neutron has a strong repulsive component of range 0.4 fm ($1\,fm = 10^{-15}\,m$) and an attractive component of range ~ 2.4 fm. The energy needed to separate the neutron from the proton in a deuterium nucleus is 2.2 MeV. Treat the neutron in deuterium as a particle of mass 1.67×10^{-27} kg in a potential well of width 2 fm. Estimate V_0 for this potential (in MeV). (ans. ≈ 66 MeV.)

8. A proton (mass = 1.67×10^{-27} kg) is bound in a finite one-dimensional square well with a width of 2.0 fm. and a depth of 40 MeV. How many bound states are there for this system? (ans. one)

9. An electron (mass = 9.1×10^{-31} kg) is bound in a finite one-dimensional square well with a width of 1.0 nm and a depth of 0.30 eV. How many bound states are there for this system? (ans. one)

10. Consider the well shown below



$V(x) = \infty, \; x \le 0,$

$V(x) = 0, \; 0 < x < a$

$V(x) = V_0, \; x \ge a$

Demonstrate that it is possible for this well to have no bound states (for a particle of mass m)

(The solutions will be given by $-\alpha \cot \alpha = (D^2 - \alpha^2)^{1/2}$)