Physics 3143 Homework # 3

Griffiths Problems 1.3, 1.4, 1.9, 1.16, 1.17

6. Suppose ψ is an eigenfunction of the operator d/dx i.e.

$$d/dx\, \psi = c\,\psi$$

If the function is periodic in 2π $\{\psi(-\pi) = \psi(\pi)\}$, determine the eigenvalues and the corresponding expressions for the eigenfunctions. Normalize them.

7. For a step potential express k_2/k_1 as a function of V_0/E and then show that

(i) For $E \gg V_0$, $T = 1$, $R = 0$, i.e. there is total transmission.

(ii) For $E = V_0$, $T = 0$, $R = 1$, i.e. there is total reflection

8. Show that an electron with $V_0 - E = 1\,\text{eV}$ can tunnel to a distance ≈ 0.1 nm into a step barrier (Hint: at what distance x is $De^{-k_2 x}$ one half of the value it has at $x = 0$)

9. For $\psi(x) = Ae^{ik_1 x} + Be^{-ik_1 x}$ show that the probability current density

$$j_x = \left(|A|^2 - |B|^2\right) k_1 \hbar / m$$

whence for the potential step problem with $E > V_0$ show that j_x in regions I (see enclosed Figure) and II is

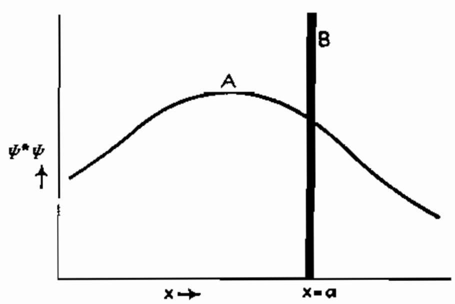
$$j_x^{I} = 4|A|^2 \hbar/m \; k_1^2 k_2 / (k_1 + k_2)^2 = j_x^{II}$$

10. Consider the average value of the dynamical function $G(p_x, x)$

$$\overline{G} = \int_{-\infty}^{+\infty} \Psi^*(x,t)\, G\left(\frac{h}{2\pi i}\frac{\partial}{\partial x}, x\right) \Psi(x,t)\, dx$$

in which the operator G, obtained from $G(p_x, x)$ by replacing p_x by $\frac{h}{2\pi i}\frac{\partial}{\partial x}$, operates on the wavefunction $\Psi(x,t)$

Consider the two types of probability distribution we discussed in class.



Two types of probability distribution function $\Psi^*\Psi$.

Note that even if the system is in a stationary state, represented by the wavefunction $\Psi_n(x,t) = \psi_n(x)\, e^{-2\pi i E_n t/h}$, only an average value can be predicted for an arbitrary dynamical quantity. Show however, that the energy of the system, corresponding to the Hamiltonian function $H(p_x, x)$ has a definite value for a stationary state (B above) of the system equal to the characteristic value, E_n, found on solution of the wave equation so that the result of a measurement of the energy of the system in a given stationary state can be predicted accurately. (Hint apply the amplitude equation to demonstrate $\overline{H^r} = (\overline{H})^r$).

11. Obtain the average position of a particle in a box of length l.