Physics 3143 Hmwk # 2A

1. Using the uncertainty principle in the form $xp = \hbar/4\pi$ show that the zero point energy of the harmonic oscillator is $\frac{1}{2}\hbar\nu$, where ν is the classical vibrational frequency $\nu = 1/2\pi\,(k/m)^{1/2}$ when the equation of motion is

$$m\frac{d^2x}{dt^2} = -kx$$

and the potential energy is $\frac{1}{2}kx^2$. Prove also that

$$\nu = 1/2\pi\,(k/m)^{1/2}$$

from the equation of motion.

2. Consider the transition of an atom initially in an excited state emitting a photon $\hbar\nu = E$ and falling to the ground state. From the equation for a wave packet $(\Delta x)(\Delta k) = 1$ show that $(\Delta E)(\Delta t) = \hbar$ (so that an appropriate form of the Uncertainty Principle for root mean square values is approximately $(\Delta E)(\Delta t) = \hbar/4\pi$.

3. A particle of mass m in a one dimensional box of length ℓ has eigenvalues of the energy $E = n^2\hbar^2/8m\ell^2$ $(n = 1, 2, 3, \ldots)$ and corresponding eigenfunctions $\psi_n(x) = (2/\ell)^{1/2}\sin(n\pi x/\ell)$, $0 \le x \le \ell$. $\psi(x) = 0$ for all other values of x.

(a) Find the average value, $\langle \cos n\pi x/\ell \rangle_n$, for a particular state n, of the function $\cos n\pi x/\ell$.

(b) Find the value of the orthogonality integral S

$$S = \int_0^\ell \psi_{n_1}(x)\,\psi_{n_2}(x)\,dx \qquad n_1 \neq n_2$$

(c) Find the value of the integral S when $n_1 = n_2$