Physics 3143 — Homework # 2   Starred problems are for the ambitious.

1. What states of polarization do the following represent (specify the answer as an angle with respect to the transmission axis of the polarizer

a) $|t\rangle + |a\rangle$ (ans. 45°)

b) $|t\rangle + 2|a\rangle$ (ans. 63°)

c) $2|t\rangle - |a\rangle$ (ans. 27°)

2. Normalize the states in problem I

(answers a). $\times 1/2^{1/2}$, $\times 1/5^{1/2}$, $\times 1/5^{1/2}$

3. Show that the states in example 1(b) and 1(c) are orthogonal to each other.

4.* Derive the Compton shift

$$\Delta\lambda = \lambda' - \lambda = \hbar/m_0c\,(1-\cos\phi)$$

you will need to obtain expressions for $\left(1/\lambda - 1/\lambda'\right)^2$ and for $(\gamma - 1)$, both from

$$\hbar c/\lambda = \hbar c/\lambda' + \gamma m_0c^2 - m_0c^2$$

5.* Show that, in the Compton effect, the relation between the electron recoil angle θ and the photon scattering angle ϕ is

$$\tan\theta = 1/(1 + \hbar\nu_0/mc^2)\,(\operatorname{ctn}(\phi/2))$$

Show that the recoiling electron has kinetic energy

$$K = E_{ph}^2 (1-\cos\phi) \Big/ m_0c^2 + E_{ph}(1-\cos\phi)$$

for an incident photon energy E_{ph}.
(consider starting with the Compton shift equation and $\lambda = c/\nu$)

6. Show that for very high energy photons, the kinetic energy of the recoiling electron asymptotically approaches

$$K = E_{ph} - 255.5 \text{ keV}.$$

for backscattering ($\phi = 180°$) (See expression in Problem 5)

7. For a photon with wavelength λ_ϕ and a particle of mass m and velocity v with de Broglie wavelength λ_{deB}; obtain a relationship between their energies for $\lambda_\phi = \lambda_{deB}$ (assume that the particle has zero potential energy). Which would be best for microscopy at length scales of a) 10^{-6} m (irises), (b) 10^{-10} m (atoms), c) 10^{-15} m (nuclei), photons or electrons, from the viewpoint of energy needs.

8. An electron is confined in an infinite square well of width $\mathcal{L}$. What value of $\mathcal{L}$ is needed if the zero-point velocity of the electron is the same as that for an electron in the ground state orbit of the hydrogen atom. Compare your answer with the radius of the n=1 Bohr orbit in hydrogen? (ans. $\mathcal{L} = 0.165$ nm).

9. For a neutron ($m = 1.67 \times 10^{-27}$ kg) confined within a nucleus (diameter $\approx 10^{-14}$ m), use the particle-in-a-box model to estimate typical excitation energies of neutrons within a nucleus. (ans. ≈ 2 MeV)

10. Calculate the probability that a particle of mass m confined in an infinitely deep potential well of width ℓ will be observed between $x = 0$ and $x = \ell/3$ ($x = 0$ is defined to be the lefthand side of the box) when it is in the $n = 2$ state. (ans. ~ 0.402)

11. Solve the particle-in-the-box problem for $V(x) = \infty$, $x \leq -L/2$; $V(x) = 0$, $-L/2 < x < +L/2$; $V(x) = \infty$, $x \geq +\ell/2$; i.e. obtain E_n and $\Psi_n(x)$.

12. At what velocity is the de Broglie wavelength of an alpha-particle equal to that of a 1-keV x-ray?

13. Show that 54 eV. electrons will be coherently diffracted at an angle of 50° for a nickel surface which has a lattice spacing of 2.15Å. You will need to recall the Bragg relationship and use de Broglies matter wave hypothesis.