

FINAL EXAMINATION OF SPRING SEMESTER 2006

Physics 2211E

Test Form **651** Name _____

Test Form = 3-digit number at the top of the first page of this test.

Student Number = Your full 9-digit GTID# (Georgia Tech ID number)

1. Print your name, test form number and student number in the section of the answer card labeled "STUDENT IDENTIFICATION".
2. Bubble in your test form number in columns 1-3, skip column 4, and bubble in your student number in columns 5-13.
3. For each question, select the answer most nearly correct, circle this answer on your test, and bubble it in on your answer card. Do not put any extra marks on the card.

There are 18 problems. The total score is 180, for 10 points each. Equation sheet is attached.

Problem 1 : A vector A is added to $B = 6i - 8j$. The resultant vector is in the positive x direction and has a magnitude equal to A. What is the magnitude of A?

- (a) 11
- (b) 5.1
- (c) 7.1
- (d) 8.3
- (e) 10

$$A = A_x i + A_y j$$

$$A_y - 8 = 0 \quad A_y = 8$$

$$A_x^2 + 8^2 = (A_x + 6)^2 \quad A_x = 7/3$$

$$A^2 = (7/3)^2 + 8^2$$

$$A = 8.3$$

Problem 2 : Two objects begin to free fall from rest from the same point 2.0 s apart. How long after the **FIRST OBJECT** begins to fall will the two objects be 40 meters apart (let $g = 10 \text{ m/s}^2$)?

- (a) 4.0 s
- (b) 1.0 s
- (c) 3.0 s
- (d) 2.0 s
- (e) 0 s

$$S_1 - S_2 = 40$$

$$0.5 g t^2 - 0.5 g (t-2)^2 = 40$$

$$5 t^2 - 5 (t-2)^2 = 40$$

$$t^2 - (t-2)^2 + 4t - 4 = 8$$

$$4t = 12$$

$$t = 3$$

Problem 3: At $t = 0$, a particle leaves the origin with a velocity of 12 m/s in the positive y direction and moves in the xy plane with a constant acceleration of $(2.0 \text{ i} - 4.0 \text{ j}) \text{ m/s}^2$. At the instant the particle moves back across the x axis ($y = 0$), what is the speed of the particle?

- (a) 17 m/s
- (b) 16 m/s
- (c) 18 m/s
- (d) 14 m/s
- (e) None of the above

$$S_y = 12 * t - 0.5 * 4 * t^2 = 0$$

$$t = 6 \text{ s}$$

$$v_y = v_{y0} = 12 \text{ m/s}$$

$$v_x = 2 t = 12$$

$$v = 17 \text{ m/s}$$

Problem 4: During volcanic eruptions, chunks of rock are blasted out of the volcano. At what initial speed must a block be ejected 45° above the horizontal so as to land at a point displaced 2.8 km below and 8.6 km horizontally from the initial point?

- (a) 0.28 km/s
- (b) 0.25 km/s
- (c) 0.18 km/s
- (d) 0.22 km/s
- (e) None of the other choices.

$$S_y = v_y * t - .5 * 9.8 * t^2 = - 2800$$

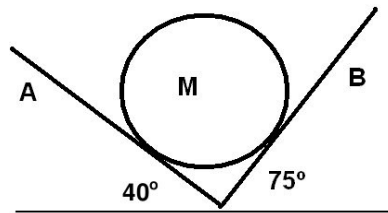
$$S_x = v_x * t = v_y * t = 8600$$

$$t = 48.2 \text{ s}$$

$$v_x = v_y = 178.3 \text{ m/s}$$

$$v = 252 \text{ m/s} = 0.25 \text{ km/s}$$

Problem 5: A uniform 3.0-kg sphere rests on two flat frictionless surfaces as shown in the figure. What is the magnitude of the force of surface B on the sphere?



- (a) 18 N
- (b) 24 N
- (c) 9.0 N
- (d) 27 N
- (e) 21 N

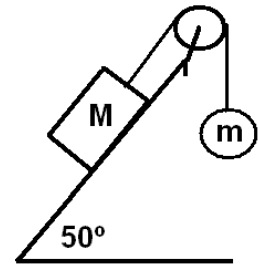
$$B \sin 75 = A \sin 40$$

$$B \cos 75 + A \cos 40 = 3g$$

$$B (\cos 75 + \cos 40 * \sin 75 / \sin 40) = 3g$$

$$B = 21 \text{ N}$$

Problem 6: In the figure shown, $M = 100 \text{ kg}$, and $m = 50 \text{ kg}$. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.20$. The pulley is frictionless. What is the magnitude of the acceleration of the block? (Hint: Will the mass m go up or down?)

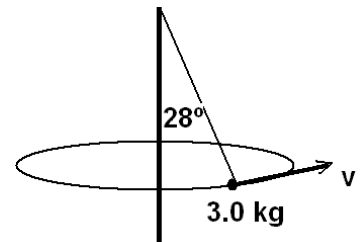


- (a) 2.6 m/s^2
- (b) 2.3 m/s^2
- (c) 1.3 m/s^2
- (d) 0.90 m/s^2
- (e) 0.75 m/s^2

$$a = (Mg \sin 50 - Mg \cos 50 \mu_k - mg) / (M + m)$$

$$= 0.90 \text{ m/s}^2$$

Problem 7: A 3.0-kg ball attached by a 2.0 m string to a vertical axis is rotated about that axis and moves in a horizontal circle. The angle between the string and the axis is 28° . What is the speed of the ball?



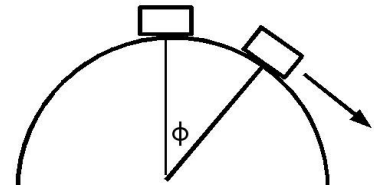
- (a) 2.8 m/s
- (b) 3.2 m/s
- (c) 4.0 m/s
- (d) 4.8 m/s
- (e) 2.2 m/s

$$F = mg \tan 28 = m v^2 / r = m v^2 / (2 \sin 28)$$

$$v^2 = g \tan 28 * 2 \sin 28 = 4.89$$

$$v = 2.21 \text{ m/s}$$

Problem 8 : A sled starts from rest at the top of the frictionless, hemispherical, snow-covered hill shown in figure. At what angle ϕ does the sled “fly off” the hill?



- (a) $\text{Arc cos } (2/3)$
- (b) $\text{Arc cos } (1/3)$
- (c) $\text{Arc cos } (1/2)$
- (d) $\text{Arc cos } (1/4)$
- (e) $\text{Arc cos } (2/5)$

$$mgh = \frac{1}{2} m v^2$$

$$v^2 = 2 g R(1 - \cos \phi)$$

$$mg \cos \phi = mv^2 / R$$

$$g \cos \phi = v^2 / R = 2 g (1 - \cos \phi)$$

$$\cos \phi = 2 (1 - \cos \phi)$$

$$3 \cos \phi = 2$$

$$\phi = \text{Arc cos } (2/3)$$

Problem 10.74

Problem 9 : Two point masses M and $9M$ are separated by a distance L . The distance from M at which there will be no net gravitational force will be:

- (a) $8L / 9$
- (b) $3L / 4$
- (c) $L / 9$
- (d) $L / 4$
- (e) $L / 10$

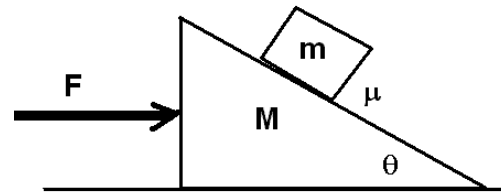
$$GMm/a^2 = G9M m / (L-a)^2$$

$$L-a = 3 a$$

$$L = 4a$$

$$a = L/4$$

Problem 10: A block of mass m and a wedge of mass M are accelerating by a horizontal force F applied to the wedge. The coefficient of static friction between them is μ . The surface between the wedge and the horizontal surface is frictionless. The angle theta is θ . What is the maximum force F so that they do not slide relative to each other?



- (a) $\frac{(M+m)g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}$
 (b) $\frac{(M+m)g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$
 (c) $\frac{(M+m)g(\mu\sin\theta + \cos\theta)}{\sin\theta - \mu\cos\theta}$
 (d) $\frac{(M+m)g(\mu\sin\theta - \cos\theta)}{\sin\theta + \mu\cos\theta}$
 (e) $\frac{(M+m)g(\mu\sin\theta + \cos\theta)}{\cos\theta - \mu\sin\theta}$

The friction is down the incline.

Normal force on m : N

$$N \cos\theta = mg + N \mu \sin\theta$$

$$N = mg / (\cos\theta - \mu \sin\theta)$$

$$a = (N \sin\theta + N \mu \cos\theta) / m$$

$$= g(\sin\theta + \mu \cos\theta) / (\cos\theta - \mu \sin\theta)$$

$$F = (M + m) a = (M + m)g(\sin\theta + \mu \cos\theta) / (\cos\theta - \mu \sin\theta)$$

Problem 11: After a completely inelastic collision, two objects of the same mass and same initial speed are found to move away together at 0.707 their initial speed. Find the angle between the initial velocities of the objects.

- (a) 120°
 (b) 45°
 (c) 60°
 (d) 90°
 (e) 30°

$$m\mathbf{v}_i + m\mathbf{v}_i = 2m\mathbf{v}_f$$

$$m v_{ix} + m v_{ix} = 2m v_{fx}$$

$$m v_{iy} - m v_{iy} = 0$$

$$v_f = v_{fx} = v_{ix}$$

$$\sin(\theta/2) = v_{ix}/v = 0.707$$

$$\theta / 2 = 45$$

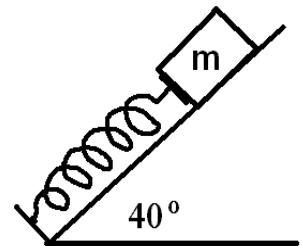
$$\theta = 90$$

Problem 12: A 5.0-kg block is pushed down a plane (inclined at 20 from the horizontal) by a force (magnitude $P = 10 \text{ N}$) acting parallel to the plane and down the plane. If the block moves with a constant speed of 1.5 m/s, at what rate is the gravitational force doing work on the block?

- (a) +69 W
- (b) +74 W
- (c) +25 W
- (d) - 25 W
- (e) zero W

speed downward : $1.5 \sin 20 = 0.513 \text{ m/s}$
 Power = $mg v = 5 * 9.8 * 0.513 = 25.1 \text{ W}$

Problem 13: A spring ($k = 800 \text{ N/m}$) is arranged on a plane inclined 40° with the horizontal as shown in the figure. A 1.5 kg mass is pushed against the spring until it is compressed 0.30 m and the system is then released from rest. If the coefficient of kinetic friction between the plane and the mass is 0.40, how far will the mass slide from the point of release before stopping momentarily?

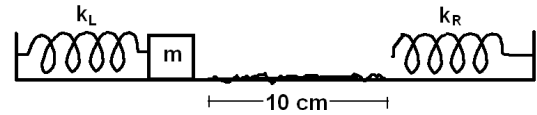


- (a) 7.0 m
- (b) 7.3 m
- (c) 2.0 m
- (d) 4.1 m
- (e) 2.6 m

$$\frac{1}{2} k x^2 = mgh + f s = mg * s * \sin 40 + mg \cos 40 * 0.40 * s$$

$$s = (\frac{1}{2} 800 (.30)^2) / (mg \sin 40 + mg \cos 40 * 0.40) = 2.58 \text{ m}$$

Problem 14: A 1.0-kg mass is free to move on a horizontal plane between two fixed springs as shown in the figure. The plane is frictionless except for a small region 10 cm long where $\mu_k = 0.25$. The mass is pressed against the left-hand spring ($k_L = 500 \text{ N/m}$) compressing it a distance x_0 and released from rest. The mass slides across the plane, strikes the right-hand spring, and rebounds back to compress the left-hand spring. When the mass comes momentarily to rest the left-hand spring is compressed by 2.0 cm. What was the initial compression x_0 ?



- (a) 5.2 cm
- (b) 4.9 cm
- (c) 6.0 cm
- (d) 6.7 cm
- (e) 7.3 cm

$$\frac{1}{2} k_L x_0^2 = mg\mu_k \cdot .20 + \frac{1}{2} k_L \cdot .02^2$$

$$x_0^2 = mg\mu_k \cdot .20 * 2 / k_L + \cdot .02^2$$

$$x_0 = 4.86 \text{ cm}$$

Problem 15: Two blocks with masses of 2.0 kg and 3.0 kg are placed on a horizontal frictionless surface. A light spring is placed in a horizontal position between the blocks. The blocks are pushed together, compressing the spring, and then released from rest. After contact with the spring ends, the 3.0-kg mass has a speed of 2.0 m/s. How much potential energy was stored in the spring when the blocks were released? (Hint: total momentum is conserved)

- (a) 15 J
- (b) 3.0 J
- (c) 6.0 J
- (d) 12 J
- (e) 9.0 J

$$PE = KE = \frac{1}{2} m v^2 + \frac{1}{2} m' v'^2$$

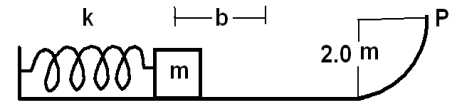
$$mv = m'v' = 3 * 2 = 6$$

$$v = 2$$

$$v' = 3$$

$$PE = \frac{1}{2} 3 * 2^2 + \frac{1}{2} 2 * 3^2 = 15 \text{ J}$$

Problem 16: A mass $m = 1.2 \text{ kg}$ is released from rest after compressing a spring ($k = 300 \text{ N/m}$) by an amount $b = 0.50 \text{ m}$. The mass slides without friction along a track that has the shape of one quarter of a circle at its end. Find the magnitude of the TOTAL acceleration of the mass at the point P.



- (a) 9.8 m/s^2
- (b) 1.9 m/s^2
- (c) 21 m/s^2
- (d) 12 m/s^2
- (e) 15 m/s^2

$$a_x = v^2/r$$

$$a_y = g$$

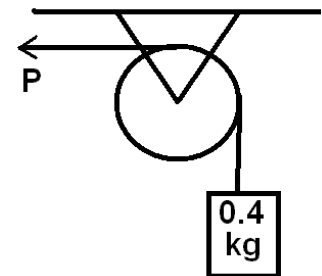
$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2 + mgh$$

$$v^2 = (kx^2 - 2mgh)/m = (300 \cdot 0.50^2 - 2 \cdot 1.2 \cdot 9.8 \cdot 2) / 1.2 = 23.3$$

$$a_x = 23.3 / 2 = 11.65$$

$$a = \sqrt{(11.65^2 + 9.8^2)} = 15.2 \text{ m/s}^2$$

Problem 17: A wheel (radius = 0.20 m) is mounted on a frictionless horizontal axis. A light cord wrapped around the wheel supports an 0.40 kg object. When a force (magnitude $P = 6.0 \text{ N}$) is applied to the wheel as shown in the figure, the 0.40 kg object has an upward acceleration of 1.2 m/s^2 . What is the moment of inertia of the wheel about the axis of rotation? (Hint: calculate the tension on the vertical cord first.)



- (a) 0.067 kg m^2
- (b) 0.053 kg m^2
- (c) 0.080 kg m^2
- (d) 0.040 kg m^2
- (e) 0.19 kg m^2

$$a = 1.2 \text{ m/s}^2.$$

$$T' = m(g + a) = 0.4 (9.8 + 1.2) = 4.4 \text{ N}$$

$$\alpha = a / r = 1.2 / .2 = 6$$

$$\text{Torque}_{\text{net}} = P r - T' r = I \alpha$$

$$I = (P r - T' r) / \alpha = (6 * 0.2 - 4.4 * 0.2) / 6 = 0.04 \text{ kg m}^2$$

Problem 18: A block on a frictionless table is connected as shown in the figure to two springs having spring constants k_1 and k_2 . Which expression for the block's oscillation frequency f in terms of the frequencies f_1 which it would oscillate if attached to spring 1 or spring 2 alone? (Hint: calculate the ratio of the extensions x_1/x_2 of the two springs when they are stretched by same force F)

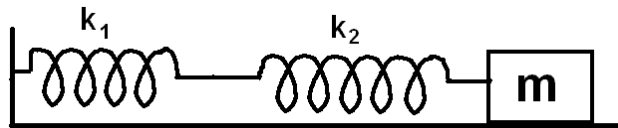


table is
springs
is the
and f_2 at

(a) $f_1 + f_2$

(b) $\sqrt{f_1^2 + f_2^2}$

(c) $\sqrt{\frac{f_1^2 f_2^2}{f_1^2 + f_2^2}}$

(d) $\frac{f_1 + f_2}{f_1 f_2}$

(e) $\frac{f_1 f_2}{f_1 + f_2}$

$$F = k_1 x_1$$

$$F = k_2 x_2 \rightarrow x_2/x_1 = k_1/k_2$$

let k be the spring constant of spring(1+2)

$$F = k (x_1 + x_2)$$

$$k_1 x_1 = k (x_1 + x_2)$$

$$k = k_1 x_1 / (x_1 + x_2) = k_1 / (1 + x_2/x_1)$$

$$\text{since } x_2/x_1 = k_1/k_2$$

$$k = k_1 / (1 + k_1/k_2) = k_1 k_2 / (k_1 + k_2)$$

$$f_1 = \sqrt{k_1/m} / 2\pi$$

$$f_2 = \sqrt{k_2/m} / 2\pi$$

$$k_1 = (2\pi f_1)^2 m$$

$$k_2 = (2\pi f_2)^2 m$$

$$k = (2\pi)^2 m f_1^2 f_2^2 / (f_1^2 + f_2^2)$$

$$f = \sqrt{k/m} / 2\pi = \sqrt{f_1^2 f_2^2 / (f_1^2 + f_2^2)}$$