

# Nuclear Magnetic Resonance

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We assemble an experiment to perform pulsed Nuclear Magnetic Resonance using an electromagnet and RF electronics. Using this device, we observe Free Induction Decay in samples of glycerine and mineral oil. Spin-Lattice relaxation times are measured for both samples using a two-pulse sequence, and Spin-Spin relaxation times are measured using the Meiboom-Gill multi-pulse sequence.

## I. INTRODUCTION TO NUCLEAR MAGNETIC RESONANCE

Nuclear Magnetic Resonance (NMR) was originally developed by I.I. Rabi in the late 1930's as a way of measuring the nuclear magnetic moment. Rabi's method involved a molecular beam of LiCl in an oscillating magnetic field. [1] Later, a far more flexible technique which could be used in solids and liquids was developed by Edward Purcell. In Purcell's method, a sample containing hydrogen (paraffin was the first tested substance) is placed in a large constant magnetic field, and a radio frequency (RF) resonator is placed around the sample. The hydrogen in the sample absorbs RF radiation, then slowly radiates it away. [2] Both Rabi and Purcell were awarded Nobel prizes for their discoveries.

In our experiment, we used an apparatus very similar to Purcell's to study Nuclear Magnetic Resonance in two liquids, mineral oil and glycerine.

## II. THE THEORY OF NMR

Nuclear Magnetic Resonance is a phenomenon related to the spin angular momentum  $I$  of the proton within a hydrogen atom (In this experiment, hydrogen is used, but NMR can work on a large variety of elements). Recall that for the proton, there are two spin states,  $\pm 1/2$ . When particles have a spin, they also have a magnetic moment,  $\mu$ :

$$\mu = \gamma I \quad (1)$$

where  $\gamma$  is the gyromagnetic ratio. The magnitude of angular momentum is quantized, as well as the direction - the magnetic number  $m$  governs this, and can take values from  $+I$  to  $-I$ , in an integral number of steps in between. The z-component of the angular momentum is

$$I_z = m\hbar \quad (2)$$

so the z-component of the magnetic moment is

$$\mu_z = \gamma m\hbar. \quad (3)$$

When you put this magnetic moment in a magnetic field  $\vec{B}$ , it gains an energy

$$E = -\vec{B}\mu = -\mu_z B = -\gamma m\hbar B \quad (4)$$

(assuming  $\vec{B}$  is oriented in the z direction.) So, there is an energy difference between the two states  $m = +\frac{1}{2}$  and  $m = -\frac{1}{2}$  equal to

$$\Delta E = \gamma m\hbar B. \quad (5)$$

This shows that the lower spin state is a little more favorable, energy-wise, so the population of atoms in the lower state is slightly larger than in the upper state. If a photon with energy equal to  $\Delta E$  hits the photon, resonant absorption occurs. The frequency of this photon is given by

$$\nu = \frac{\Delta E}{h} = \frac{\gamma B}{2\pi}. \quad (6)$$

When a sample is bombarded by photons at the resonant frequency, many of the atoms will jump to the higher state, then slowly 'relax' back into the lower, thermodynamically favorable state. The average time it takes for this process to occur is the relaxation time  $T_1$  for the sample. In other words,  $T_1$  is the decay constant for the z-component magnetization. This is often called Spin-Lattice Relaxation.

In addition to  $T_1$ , there is another decay time which measures the decay of magnetization in the  $xy$  plane. This  $xy$  magnetization decays because each spin experiences a slightly different magnetic field due to random fluctuations, so each spin decays at a slightly different rate. This leads to a loss of coherence, which means no net magnetization in the  $xy$  plane. This is often called Spin-Spin Relaxation. Both  $T_1$  and  $T_2$  depend on the sampled material, which is the basis for Magnetic Resonance Imaging (MRI).

## III. THE EXPERIMENT

### A. Equipment

Our experimental setup is was designed to measure the decay times  $T_1$  and  $T_2$ . A large electromagnet in a Helmholtz configuration supplies a homogenous magnetic

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field of around .36 Tesla. In the homogenous field, a sample holder holds a vial inside two coils. The first coil is a transmitting coil, which is wound around the sample vial with an axis perpendicular to the magnetic field. The second coil is a receiver, which is wound perpendicular to both the field, and the transmitting coil. The receiver measures magnetization in the  $xy$  plane (assuming the magnetic field direction to be the  $z$  direction) by induction - as the magnetization of the atoms change, the field changes, which induces a current in the coil.

Both the receiving and transmitting coils are connected to the 'TeachSpin PS1-A,' which is a pre-assembled box containing an RF oscillator and amplifier (the transmission electronics), a pulse programmer (which allows you to set up custom pulse lengths and patterns), and an RF mixer and amplifier (the receiving electronics). The output from the mixer and amplifier are connected to the two channels of an oscilloscope, which lets us view the free induction decay signal. The oscilloscope connects to the computer, so that the traces can be saved for later analysis.

## B. Procedure

Our procedure was to fire a sequence of pulses of RF radiation at the sample, which would excite the sample. Then, the sample would slowly decay back into thermal equilibrium. This decay was recorded on the oscilloscope screen. By using different pulse sequences, we could measure two properties: The aforementioned  $T_1$  and  $T_2$ . We measured these values for two different samples: mineral oil, and glycerine.

### 1. Observing a Free Induction Decay Signal

The first pulse sequence was just a single pulse, called a  $\frac{\pi}{2}$  pulse. The  $\frac{\pi}{2}$  pulse can be thought of as rotating the magnetization (initially in the  $z$  direction) of the sample by  $90^\circ$  into the  $xy$  plane. The magnetization then slowly precesses to its equilibrium state.

We recorded the decay of the  $xy$  magnetization as a function of time using the oscilloscope.

### 2. Measuring $T_1$

The second pulse sequence was a  $\pi$  pulse (which rotates the magnetization by  $180^\circ$ ), followed by a  $\frac{\pi}{2}$  at some delay time later. After the  $\pi$  pulse, the magnetization starts to decay back to its original state. The  $\frac{\pi}{2}$  pulse lets you measure this by rotating the magnetization into the  $xy$  plane. Of interest is the delay time at which you observe no  $xy$  magnetization, as this is the point at which the magnetization crosses zero, and is equivalent to the half-life of the exponential decay back into its original state.

We varied the delay time until we found the point at which no magnetization occurs, and recorded it.

### 3. Measuring $T_2$

The final pulse sequence is the most complex: it is a  $\frac{\pi}{2}$  pulse, followed by a long series of  $\pi$  pulses. These  $\pi$  pulses create 'spin echo' signals, which let you measure the decay which occurs because of interactions between each magnetic moment and its neighbors. This is spin-spin relaxation.

We used a series of 99  $\pi$  pulses, so we had 99 spin echoes. Each echo is less the echo before it, since the magnetization is slowly decaying after the  $\frac{\pi}{2}$  pulse. We recorded this series of decaying echoes as a function of time.

## IV. DATA AND ANALYSIS

Since the decay times measured in this lab are dependent on many environmental factors, including magnetic field strength, magnetic field homogeneity, temperature, and sample purity, there are no easy ways to compare the measurements against a prediction. This does not mean that nothing useful can be inferred - the decay times are different for different materials, and can be used for identification purposes (the bases of magnetic resonance imaging, or MRI). In the analysis of this data, closeness of fit to an exponential decay curve is calculated. This determines how closely our data agrees with the theory that the magnetization decays exponentially back to its equilibrium state.

### A. Calculating $T_1$

From the recorded zero-point delay time ( $\Delta T$ ), which is the half-life of the decay process,  $T_1$  can be calculated by the simple formula

$$T_1 = \frac{\Delta T}{\ln 2}. \quad (7)$$

### B. Calculating $T_2$

The recorded data from the multiple pulse sequence is shown in figure 1. The calculation was as follows: First, a Mathematica program extracted the peaks from the data. The log of the peak data was taken, and regression was used to fit these points to a straight line. For a regression line  $y = ax + b$ ,  $T_2$  can be calculated by finding  $\frac{-1}{a}$ .

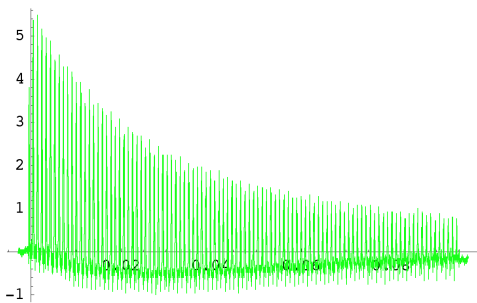


FIG. 1: The data recorded from the measurement of  $T_2$  for mineral oil. Each of the sharp peaks is a spin echo.

## V. CONCLUSIONS

### A. Mineral Oil

Using the above calculation methods, we found  $T_1 = 42.0ms$  and  $T_2 = 49.6ms$  for mineral oil.

The mineral oil T2 data agrees with the theory. The adjusted r-squared value for the fit line is 0.966703, which means the data fits an exponential decay somewhat well. However, an examination of the fit line residuals (see figure 2) suggest that there may be a sine wave component to our recorded data. This suggests that we were slightly off resonance with the atoms when we performed the measurement (we had observed several times that if you were further off resonance, you could see a very clear beat pattern on top of the spin echo decay signal). This is likely due to the fact that over the course of running the experiment, the magnet heats up, which alters the field strength, which changes the resonance frequency (equation 6).

### B. Glycerine

We calculated  $T_1 = 85.7ms$  and  $T_2 = 58.0ms$  for glycerine.

The glycerine T2 data agreed very well with the theory. The adjusted r-squared value for the fit line (0.995059) suggests that the data is extremely close to an exponential decay. Figure 3 shows the fit of the data, as well as a plot of the residuals.

### C. Final Thoughts

We have demonstrated that the experimental apparatus can produce useful measurements of the decay times for liquid materials. While the subject is theoretically fairly complex, this easy-to-perform experiment helps give a solid understanding of the material. The experiment is fairly reliable - the only component which causes trouble are the magnets, which are not very homogenous, and heat up over time.

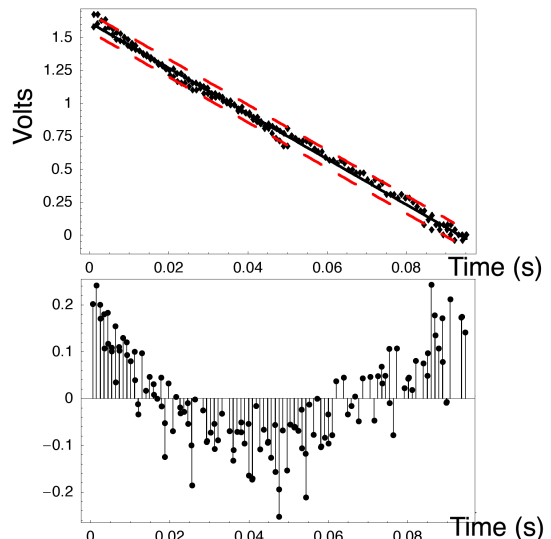


FIG. 2: Plot of the log of our spin echo peaks as a function of time. A linear fit of  $y = 1.48957 - 20.1424x$  was found (solid black line), with adjusted  $R^2 = 0.966703$ . Also drawn are the 95 percent confidence bounds (dashed red lines). Below is a plot of the residuals for the fit line.

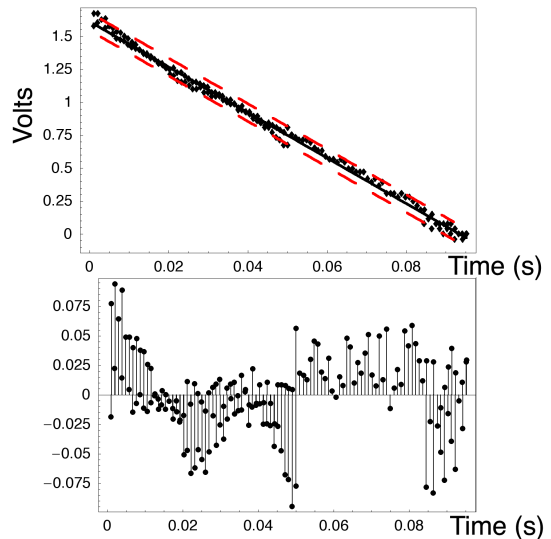


FIG. 3: Plot of the log of our spin echo peaks as a function of time. A linear fit of  $y = 1.61281 - 17.2512x$  was found (solid black line), with adjusted  $R^2 = 0.995059$ . Also drawn are the 95 percent confidence bounds (dashed red lines). Below is a plot of the residuals for the fit line. The residuals show the same oscillations as observed in mineral oil, but not nearly as pronounced.

The apparatus is very flexible, hence there are several ways the experiment could be expanded - decay times could be measured as a function of sample dilution, additional samples could be added, and the Fourier Transform of the free induction decay signal could be performed and analyzed.

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- [1] Rabi, I. I. and Zacharias, J. R. and Millman, S. and Kusch, P., A New Method of Measuring Nuclear Magnetic Moment, *Phys. Rev.* 53, Pages 318-318. Feb. 1938.
  - [2] Purcell, E. M. and Torrey, H. C. and Pound, R. V., Resonance Absorption by Nuclear Magnetic Moments in a Solid. *Phys. Rev.* 69, Pages 37-38. Jan. 1946.

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